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Miniaturized Tables of Bessel Functions. III*

By Yudell L. Luke

Abstract. After the manner of our previous studies, coefficients for the expansion of $J_\nu(z)$ and $Y_\nu(z)$ in double series of Chebyshev polynomials are presented. For $J_\nu(z)$, the ranges are (1) $0 < z \leqq 8$, $0 \leqq \nu \leqq 4$, (2) $0 < z \leqq 8$, $4 \leqq \nu \leqq 8$. For $J_\nu(z) + iY_\nu(z)$, the ranges are $z \geqq 5$ and $0 \leqq \nu \leqq 1$. The coefficients are given with sufficient accuracy to enable the evaluation of the Bessel functions to at least 20 decimals.

1. Introduction. In previous studies [1], [2], we considered the expansion of two parameter functions in a double series of Chebyshev polynomials and developed coefficients for the evaluation of $K_\nu(z)$ and $I_\nu(z)$ over a large part of the real z and ν lines. In the present paper, we give similar type coefficients for the evaluation of $J_\nu(z)$ and $Y_\nu(z)$.

2. Chebyshev Expansion for $J_\nu(z)$. From [3, Vol. 1, p. 212 and Vol. 2, p. 35],

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu, \lambda) T_{2k}(z/\lambda), \qquad 0 < z \leqq \lambda, \tag{1}$$

$$A_k(\nu, \lambda) = G_k(\nu, \lambda)/2^\nu \Gamma(\nu + 1), \tag{2}$$

$$G_k(\nu, \lambda) = \frac{\epsilon_k (-)^k \lambda^{2k} \Gamma(\nu + 1)}{2^{4k} k!\ \Gamma(k + \nu + 1)}\, {}_1F_2\left(\begin{matrix} \frac{1}{2} + k \\ 1 + 2k, \nu + 1 + k \end{matrix} \middle| -\lambda^2/4\right), \tag{3}$$

$$\frac{2G_k(\nu, \lambda)}{\epsilon_k} = \frac{(k+1)}{(k+2)} \{G_{k+1}(\nu, \lambda) - G_{k+3}(\nu, \lambda)\} - \frac{16(k+1)(k+\nu+1)}{\lambda^2} G_{k+1}(\nu, \lambda) + \left\{1 - \frac{16(k+1)(k+2-\nu)}{\lambda^2}\right\} G_{k+2}(\nu, \lambda), \tag{4}$$

where

$$\epsilon_0 = 1, \qquad \epsilon_k = 2 \quad \text{for } k > 0. \tag{5}$$

It is readily shown that

$$G_k(\nu, \lambda) = \frac{\epsilon_k (-)^k \lambda^{2k} k^{-\nu}}{2^{4k} (k!)^2} [1 + O(k^{-1})], \tag{6}$$

and for ν and λ fixed,

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$$\lim_{k\to\infty} G_k(\nu, \lambda) = 0. \tag{7}$$

Thus, the expansion (1) converges and by letting $z \to 0$, we have the useful normalization relation

$$\sum_{k=0}^{\infty} (-)^k A_k(\nu, \lambda) = 1. \tag{8}$$

Further, after the manner of the discussion presented in [3, Vol. 2, pp. 159-166], we can show that use of the recursion formula (4) in the backward direction is convergent. Thus, for a fixed λ, we can generate the coefficients $A_k(\nu, \lambda)$ for any given value of ν. Then, following the discussion given in [1], we can develop coefficients $D_{r,k}(\lambda)$ such that

$$A_k(\nu, \lambda) = \sum_{r=0}^{\infty} D_{r,k}(\lambda) T_r^*\left(\frac{\nu - s}{t}\right), \qquad s \leqq \nu \leqq s + t. \tag{9}$$

We remark that 20 decimal values of $A_k(\nu, \lambda)$ are given in [3, pp. 331, 332, 352–356] for $\lambda = 8$ and $\nu = 0, \pm\frac{1}{4}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm\frac{3}{4}, 1$. Coefficients for the evaluation of $Y_0(z)$ and $Y_1(z)$ for $0 < z \leqq 8$ are also given in [3, pp. 331, 332].

We next present a descending type expansion in series of Chebyshev polynomials for the evaluation of $J_\nu(z)$ and $Y_\nu(z)$ in the vicinity of $z = +\infty$. Now,

$$H_\nu^{(1)}(z) = -\frac{2i}{\pi} e^{-i\nu\pi/2} K_\nu(ze^{-i\pi/2}), \tag{10}$$

and from [1], we have

$$K_\nu(z) = (\pi/2z)^{1/2} e^{-z} \sum_{k=0}^{\infty} G_k(\nu, \lambda) T_k^*(\lambda/z), \ \lambda \text{ fixed}, \ \lambda/z \leqq 1, \ |\arg z| < 3\pi/2. \tag{11}$$

The recursion formula for $G_k(\nu, \lambda)$ and other properties of these coefficients are given in [1]. If we write

$$\begin{aligned} H_\nu^{(1)}(z) &= J_\nu(z) + i\, Y_\nu(z) \\ &= (2/\pi z)^{1/2} e^{i(z - \nu\pi/2 - \pi/4)} \sum_{k=0}^{\infty} H_k(\nu, \lambda) T_k^*(\lambda/z), \qquad z \geqq \lambda, \end{aligned} \tag{12}$$

then the recurrence formula and other properties of the coefficients $H_k(\nu, \lambda)$ follow from those for $G_k(\nu, \lambda)$ upon replacing λ by $\lambda e^{-i\pi/2}$. We have the normalization relation

$$\sum_{k=0}^{\infty} (-)^k H_k(\nu, \lambda) = 1 \tag{13}$$

and from [1], use of the backward recurrence relation for $H_k(\nu, \lambda)$ is convergent provided $|\arg \lambda| < \pi/2$.

3. Numerical Results. From (1) and (9) with a slight change of notation, we have

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/8), \qquad 0 < z \leqq 8, \tag{14}$$

$$A_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} T_r^*\left(\frac{\nu - s}{t}\right), \qquad s \leqq \nu \leqq s + t. \tag{15}$$

In Tables 1 and 2, in the microfiche section of this issue, we present values of $D_{r,k}$ which were computed by the technique depicted in [1] for $s = 0$, $t = 4$ and $s = t = 4$, respectively. Values of $\Gamma(\nu + 1)$ required in the numerics were obtained by use of the schema of my previous paper [4]. Numerous checks were made on the coefficients. They are of the type previously discussed in [1], [2] and we dispense with further details. The computations were designed so that the coefficients for $0 \leqq \nu \leqq 4$ are accurate to about 25D while those for $4 \leqq \nu \leqq 8$ are accurate to about 27D. To evaluate $J_\nu(z)$, we must incorporate the value of z^ν. As $0 \leqq z \leqq 8$, we see that the coefficients are sufficiently accurate to produce $J_\nu(z)$ to about 20 decimals at least.

Now,

$$Y_\nu(z) = (\csc \nu\pi)[(\cos \nu\pi) J_\nu(z) - J_{-\nu}(z)] \tag{16}$$

and both $J_\nu(z)$ and $Y_\nu(z)$ satisfy the same recurrence formula

$$J_{\nu+1}(z) + J_{\nu-1}(z) = (2\nu/z) J_\nu(z). \tag{17}$$

Further, the recurrence formula for $J_\nu(z)$ is always stable in the backward direction, but only conditionally stable in the forward direction. On the other hand, the recurrence formula for $Y_\nu(z)$ is always stable in the forward direction. Thus, with the aid of the coefficients just described and the recurrence formulas, we can evaluate $Y_\nu(z)$ for all z such that $0 \leqq z \leqq 8$ and all $\nu > 0$, ν an integer excepted. We have already referred to the availability of coefficients to compute $Y_0(z)$ and $Y_1(z)$. These together with the recurrence formula for $Y_\nu(z)$ can be used to generate values of $Y_n(z)$, $n = 2, 3, \cdots$. As use of the recurrence formula in the forward direction for $J_\nu(z)$ is limited, we leave for a future paper the development of Chebyshev coefficients for $0 \leqq z \leqq 8$ and $\nu > 8$.

Using (12) with a slight change of notation, we write

$$J_\nu(z) + i\, Y_\nu(z) = (2/\pi z)^{1/2} e^{i(z - \nu\pi/2 - \pi/4)} \sum_{k=0}^{\infty} H_k(\nu) T_k^*(5/z), \qquad z \geqq 5. \tag{18}$$

Let

$$H_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu), \qquad 0 \leqq \nu \leqq 1. \tag{19}$$

Table 3, also in the microfiche section of this issue, lists values of the real and imaginary parts of $E_{r,k}$. These were obtained and checked by the methods previously described and we omit further details. The coefficients are sufficiently accurate to produce values of $J_\nu(z)$ and $Y_\nu(z)$ for ν and z as noted to about 25 decimals. Since

$$Y_{-\nu}(z) = (\cos \nu\pi) Y_\nu(z) + (\sin \nu\pi) J_\nu(z), \tag{20}$$

$$J_{-\nu}(z) = (\cos \nu\pi) J_\nu(z) - (\sin \nu\pi) Y_\nu(z), \tag{21}$$

the coefficients $E_{r,k}$ together with the recurrence formula for $Y_\nu(z)$ enable the evaluation of $Y_\nu(z)$ for all $\nu > 0$ and $z \geqq 5$. A like statement cannot be made for $J_\nu(z)$ as use of the recurrence formula in the forward direction for $J_\nu(z)$ is limited. We defer

the development of coefficients to facilitate the evaluation of $J_\nu(z)$ when both ν and z are large to a later paper.

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Mathematics Department
University of Missouri
Kansas City, Missouri 64110

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TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/8) , \quad 0 < z \leq 8$$

$$A_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} T_r^*(\nu/8) , \quad 0 \leq \nu \leq 4$$

$D_{r,k}$, $k = 0$

r					
0	0.03344	76826	75854	29055	59466
1	-0.05800	43203	03003	43320	94681
2	0.03681	64100	13250	52314	58786
3	-0.01981	64100	04210	39706	73019
4	0.00723	98645	30095	28075	67000
5	-0.00202	30322	67010	66068	57586
6	0.00038	29994	20001	03070	10078
7	0.00000	27732	83915	13615	61004
8	-0.00002	82007	49158	12350	71712
9	0.00001	20263	40599	63726	61109
10	-0.00000	32203	76503	33032	30420
11	0.00000	00107	12709	38949	04796
12	-0.00000	00761	67537	40180	44010
13	0.00000	00009	40069	90884	47392
14	0.00000	00022	44472	04327	45010
15	-0.00000	00000	06492	61414	77567
16	0.00000	00001	20145	00411	72854
17	-0.00000	00000	16417	98221	98427
18	0.00000	00000	01147	05827	62020
19	0.00000	00000	00078	33090	18661
20	-0.00000	00000	00041	64125	35750
21	0.00000	00000	00007	94203	00829
22	-0.00000	00000	00001	00061	04053
23	0.00000	00000	00000	00017	99517
24	-0.00000	00000	00000	00007	94484
25	-0.00000	00000	00000	00000	59936
26	0.00000	00000	00000	00020	36044
27	-0.00000	00000	00000	00002	98910
28	0.00000	00000	00000	00000	28471
29	-0.00000	00000	00000	00000	00401
30	-0.00000	00000	00000	00000	00111
31	0.00000	00000	00000	00000	00026
32	-0.00000	00000	00000	00000	00003

$D_{r,k}$, $k = 1$

r					
0	-0.00002	79000	05110	20743	20252
1	0.01206	04215	49007	96700	49045
2	-0.00036	03422	94454	80133	53788
3	0.00460	04264	81002	49047	02746
4	-0.00204	16859	11758	25729	93074
5	0.00073	17364	00978	29764	59744
6	-0.00021	07903	[illegible]	49410	20472
7	0.00006	72330	[illegible]	00796	97000
8	-0.00000	73[illegible]	[illegible]	26405	44706
9	0.00000	03109	[illegible]0	14000	87727
10	0.00000	02407	90303	91347	34158
11	-0.00000	01062	80373	31840	19056
12	0.00000	00266	67197	70530	42454
13	-0.00000	00048	60669	79951	77307
14	0.00000	00006	34998	34456	05405
15	-0.00000	00000	63471	44752	44697
16	-0.00000	00000	04766	98896	22044
17	0.00000	00000	07310	55619	29468
18	-0.00000	00000	00476	96283	61405
19	0.00000	00000	00067	07974	37198
20	-0.00000	00000	00006	69216	75334
21	0.00000	00000	00000	28039	94229
22	-0.00000	00000	00000	05042	97000
23	0.00000	00000	00000	01496	76328
24	-0.00000	00000	00000	00220	53005
25	0.00000	00000	00000	00026	12507
26	-0.00000	00000	00000	00001	59959
27	0.00000	00000	00000	00000	00354
28	-0.00000	00000	00000	00000	01951
29	0.00000	00000	00000	00000	00139
30	-0.00000	00000	00000	00000	00037
31	0.00000	00000	00000	00000	00003

$D_{r,k}$, $k = 2$

r					
0	0.00000	59506	17199	05754	22879
1	-0.00160	00143	45010	00408	03440
2	0.00114	79883	09596	23005	02643
3	-0.00066	82102	70452	99604	20600
4	0.00032	03275	95977	77799	21634
5	-0.00012	78095	37377	00033	00979
6	0.00004	20563	00457	31784	04588
7	-0.00001	20739	57051	30434	03428
8	0.00000	28300	40111	10203	44007
9	-0.00000	05417	44088	08279	40951
10	0.00000	00773	25152	70772	07751
11	-0.00000	00095	54763	21401	76750
12	0.00000	00009	78319	10066	65210
13	-0.00000	00004	99505	07044	46034
14	0.00000	00001	27062	10012	07679
15	-0.00000	00000	21771	30765	17202
16	0.00000	00000	02022	07010	99521
17	-0.00000	00000	00267	71410	13116
18	0.00000	00000	00005	66591	07022
19	-0.00000	00000	00003	02001	90456
20	0.00000	00000	00001	00440	66592
21	-0.00000	00000	00000	15773	03707
22	0.00000	00000	00000	01792	41263
23	-0.00000	00000	00000	00137	27584
24	0.00000	00000	00000	00007	43601
25	-0.00000	00000	00000	00001	34372
26	0.00000	00000	00000	00000	28494
27	-0.00000	00000	00000	00000	03446
28	0.00000	00000	00000	00000	00131
29	-0.00000	00000	00000	00000	00019

$D_{r,k}$, $k = 3$

r					
0	-0.00000	25051	04598	76169	30719
1	0.00014	05819	93639	35007	72890
2	-0.00010	09007	24221	05303	04931
3	0.00008	58517	11000	76423	97503
4	-0.00003	32007	81401	92137	01700
5	0.00001	41557	22271	98403	00070
6	-0.00000	51030	01528	04566	98403
7	0.00000	16151	16958	70107	99047
8	-0.00000	04375	36961	29756	47147
9	0.00000	01024	29963	00507	87091
10	-0.00000	00205	40553	00205	00397
11	0.00000	00034	00036	00500	10571
12	-0.00000	00004	63723	35213	40868
13	0.00000	00000	60019	05275	60855
14	0.00000	00000	01010	00402	53611
15	-0.00000	00000	01321	43102	59043
16	0.00000	00000	00137	30596	34681
17	-0.00000	00000	00050	36194	30917
18	0.00000	00000	00008	05977	21698
19	-0.00000	00000	00001	07469	91394
20	0.00000	00000	00000	05082	00819
21	0.00000	00000	00000	00709	69078
22	-0.00000	00000	00000	00121	81956
23	0.00000	00000	00000	00022	19368
24	-0.00000	00000	00000	00002	80603
25	0.00000	00000	00000	00000	26794
26	-0.00000	00000	00000	00000	01584
27	-0.00000	00000	00000	00000	00410
28	0.00000	00000	00000	00000	00019
29	-0.00000	00000	00000	00000	00001

TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_0(s) = s^{\nu} \sum_{k=0}^{\infty} A_k(\nu) I_{\nu+k}(s/\theta) , \quad 0 \le s < \theta ,$$

$$A_k(\nu) = \sum_{r=0}^{\infty} \Gamma_{r,k} \nu^r(\nu+1) , \quad 0 \le \nu < 1$$

$\Gamma_{r,k}$: $k = 0$

r					
0	0.00000	5A479	14733	29828	7481A
1	-0.00001	07389	18482	72994	65962
2	0.00000	76132	32384	10787	03824
3	-0.00000	47389	29944	53910	47840
4	0.00000	24788	41236	44335	70000
5	-0.00000	11081	36844	52872	48170
6	0.00000	04257	78948	56753	03914
7	-0.00000	01426	92284	07488	01931
8	0.00000	00419	34920	24172	69599
9	-0.00000	00108	68197	96884	91578
10	0.00000	00024	84451	89749	63994
11	-0.00000	00005	01197	43926	15861
12	0.00000	00000	88419	47614	41153
13	-0.00000	00000	13587	91868	60983
14	0.00000	00000	01789	82677	56758
15	-0.00000	00000	00198	87384	32942
16	0.00000	00000	00009	13845	07975
17	0.00000	00000	00001	93823	97888
18	-0.00000	00000	00000	87884	28178
19	0.00000	00000	00000	10388	02467
20	-0.00000	00000	00000	01425	71367
21	0.00000	00000	00000	00156	47963
22	-0.00000	00000	00000	00012	26956
23	0.00000	00000	00000	00000	42284
24	0.00000	00000	00000	00000	07327
25	-0.00000	00000	00000	00000	01926
26	0.00000	00000	00000	00000	00219
27	-0.00000	00000	00000	00000	00020
28	0.00000	00000	00000	00000	00002

$\Gamma_{r,k}$: $k = 0$

r					
0	-0.00000	02994	11198	13824	00335
1	0.00000	05448	56903	82788	51931
2	-0.00000	04119	42368	97753	78410
3	0.00000	02011	94672	02818	59581
4	-0.00000	01489	26284	65581	00484
5	0.00000	00449	24884	27844	09898
6	-0.00000	00268	44761	82693	68968
7	0.00000	00081	66821	42987	08849
8	-0.00000	00028	47484	13489	47588
9	0.00000	00007	87488	94812	48227
10	-0.00000	00001	94844	78788	39957
11	0.00000	00000	43147	78222	84538
12	-0.00000	00000	08584	37815	22148
13	0.00000	00000	01531	55553	28897
14	-0.00000	00000	00243	91832	35218
15	0.00000	00000	00034	88424	10748
16	-0.00000	00000	00004	88588	13532
17	0.00000	00000	00000	39883	87812
18	-0.00000	00000	00000	02279	11861
19	-0.00000	00000	00000	00117	78855
20	0.00000	00000	00000	00062	78444
21	-0.00000	00000	00000	00012	87429
22	0.00000	00000	00000	00001	69296
23	-0.00000	00000	00000	00000	18978
24	0.00000	00000	00000	00000	01603
25	-0.00000	00000	00000	00000	00101
26	0.00000	00000	00000	00000	00001
27	-0.00000	00000	00000	00000	00001

$\Gamma_{r,k}$: $k = 11$

r					
0	0.00000	00124	74025	07942	37187
1	-0.00000	00231	50477	44785	36726
2	0.00000	00174	93165	29726	17292
3	-0.00000	00114	10887	75780	60377
4	0.00000	00067	79759	79881	04699
5	-0.00000	00029	92826	02666	06479
6	0.00000	00017	34147	20174	53199
7	-0.00000	00004	51303	85788	98823
8	0.00000	00001	44171	89831	51985
9	-0.00000	00000	47471	91310	29159
10	0.00000	00000	11889	44850	07585
11	-0.00000	00000	02417	96292	98867
12	0.00000	00000	00580	38481	20668
13	-0.00000	00000	00109	43645	47688
14	0.00000	00000	00019	29113	25881
15	-0.00000	00000	00003	09869	78789
16	0.00000	00000	00000	66985	99983
17	-0.00000	00000	00000	09866	68721
18	0.00000	00000	00000	00667	12955
19	-0.00000	00000	00000	00041	96191
20	0.00000	00000	00000	00004	57866
21	-0.00000	00000	00000	00000	09179
22	-0.00000	00000	00000	00000	06871
23	0.00000	00000	00000	00000	00947
24	-0.00000	00000	00000	00000	00139
25	0.00000	00000	00000	00000	00018
26	-0.00000	00000	00000	00000	00002

$\Gamma_{r,k}$: $k = 11$

r					
0	-0.00000	00104	38882	39315	03875
1	0.00000	00088	53961	86127	49872
2	-0.00000	00084	19915	44885	43434
3	0.00000	00060	65555	69957	18584
4	-0.00000	00047	23487	23888	58104
5	0.00000	00031	10521	93422	34801
6	-0.00000	00018	47839	95426	28582
7	0.00000	00008	17781	75754	51965
8	-0.00000	00002	85939	40123	86164
9	0.00000	00000	81789	74984	39364
10	-0.00000	00000	09487	37242	27171
11	0.00000	00000	00178	95283	27913
12	-0.00000	00000	00027	28847	43478
13	0.00000	00000	00005	41688	43141
14	-0.00000	00000	00001	06389	98296
15	0.00000	00000	00000	10914	73893
16	-0.00000	00000	00000	02958	95653
17	0.00000	00000	00000	00434	14764
18	-0.00000	00000	00000	00058	21414
19	0.00000	00000	00000	00007	08184
20	-0.00000	00000	00000	00000	77137
21	0.00000	00000	00000	00000	07107
22	-0.00000	00000	00000	00000	00961
23	0.00000	00000	00000	00000	00127
24	-0.00000	00000	00000	00000	00001

TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/8) , \quad 0 \le z \le 8 ,$$

$$A_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} T_r^*(\nu/4) , \quad 0 \le \nu \le 8$$

$D_{r,k}$, $k = 12$

r					
0	0.00000	00000	17044	82531	90180
1	-0.00000	00000	21362	91272	62200
2	0.00000	00000	19075	81726	04586
3	-0.00000	00000	11960	70070	72757
4	0.00000	00000	04819	49335	79035
5	-0.00000	00000	01379	04360	30825
6	0.00000	00000	01470	05453	76555
7	-0.00000	00000	00947	70700	09320
8	0.00000	00000	00195	91781	72050
9	-0.00000	00000	00040	00530	35817
10	0.00000	00000	00017	15960	05301
11	-0.00000	00000	00004	40632	03002
12	0.00000	00000	00001	03510	36526
13	-0.00000	00000	00000	22360	70210
14	0.00000	00000	00000	04452	99716
15	-0.00000	00000	00000	00819	51659
16	0.00000	00000	00000	00139	02300
17	-0.00000	00000	00000	00022	04204
18	0.00000	00000	00000	00003	22429
19	-0.00000	00000	00000	00000	43650
20	0.00000	00000	00000	00000	05493
21	-0.00000	00000	00000	00000	00635
22	0.00000	00000	00000	00000	00065
23	-0.00000	00000	00000	00000	00006

$D_{r,k}$, $k = 13$

r					
0	-0.00000	00000	00309	91403	93200
1	0.00000	00000	00540	84703	40134
2	-0.00000	00000	00445	69127	82041
3	0.00000	00000	00290	21226	49089
4	-0.00000	00000	00172	28575	39773
5	0.00000	00000	00096	08860	26554
6	-0.00000	00000	00050	52411	72864
7	0.00000	00000	00025	19566	49313
8	-0.00000	00000	00005	37115	73135
9	0.00000	00000	00001	71376	10534
10	-0.00000	00000	00000	49871	66268
11	0.00000	00000	00000	13151	03587
12	-0.00000	00000	00000	03196	79321
13	0.00000	00000	00000	00715	53174
14	-0.00000	00000	00000	00148	29116
15	0.00000	00000	00000	00028	68327
16	-0.00000	00000	00000	00005	00817
17	0.00000	00000	00000	00000	84447
18	-0.00000	00000	00000	00000	13140
19	0.00000	00000	00000	00000	01901
20	-0.00000	00000	00000	00000	00257
21	0.00000	00000	00000	00000	00032
22	-0.00000	00000	00000	00000	00004

$D_{r,k}$, $k = 14$

r					
0	0.00000	00000	00006	49114	58324
1	-0.00000	00000	00011	97691	11666
2	0.00000	00000	00009	41983	79957
3	-0.00000	00000	00006	36893	31251
4	0.00000	00000	00003	71972	77371
5	-0.00000	00000	00001	90104	42572
6	0.00000	00000	00000	85044	22224
7	-0.00000	00000	00000	36907	38480
8	0.00000	00000	00000	12653	66925
9	-0.00000	00000	00000	04066	13647
10	0.00000	00000	00000	01207	09936
11	-0.00000	00000	00000	00328	87698
12	0.00000	00000	00000	00081	99888
13	-0.00000	00000	00000	00018	92661
14	0.00000	00000	00000	00004	04935
15	-0.00000	00000	00000	00000	88560
16	0.00000	00000	00000	00000	16064
17	-0.00000	00000	00000	00000	02991
18	0.00000	00000	00000	00000	00470
19	-0.00000	00000	00000	00000	00064
20	0.00000	00000	00000	00000	00009
21	-0.00000	00000	00000	00000	00001

$D_{r,k}$, $k = 15$

r					
0	-0.00000	00000	00000	11812	41111
1	0.00000	00000	00000	21829	61476
2	-0.00000	00000	00000	17240	49644
3	0.00000	00000	00000	11750	99668
4	-0.00000	00000	00000	06966	18997
5	0.00000	00000	00000	03598	18098
6	-0.00000	00000	00000	01668	56096
7	0.00000	00000	00000	00673	86761
8	-0.00000	00000	00000	00267	69557
9	0.00000	00000	00000	00097	45867
10	-0.00000	00000	00000	00035	01682
11	0.00000	00000	00000	00008	99888
12	-0.00000	00000	00000	00001	78177
13	0.00000	00000	00000	00000	47184
14	-0.00000	00000	00000	00000	09778
15	0.00000	00000	00000	00000	01901
16	-0.00000	00000	00000	00000	00164
17	0.00000	00000	00000	00000	00045
18	-0.00000	00000	00000	00000	00011
19	0.00000	00000	00000	00000	00002

TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_\nu(x) = x^\nu \sum_{n=0}^{\infty} A_n(\nu) T_n^*(x/8) , \quad 0 \le x \le 8 ,$$

$$A_n(\nu) = \sum_{r=0}^{\infty} C_{r,n} T_r^*(\nu/4) , \quad 0 \le \nu \le 4$$

r	$C_{r,n}$, n = 16
0	0.00000 00000 00000 00100 33937
1	-0.00000 00000 00000 00340 59763
2	0.00000 00000 00000 00276 90515
3	-0.00000 00000 00000 00189 09779
4	0.00000 00000 00000 00113 33030
5	-0.00000 00000 00000 00059 40091
6	0.00000 00000 00000 00027 58300
7	-0.00000 00000 00000 00011 44475
8	0.00000 00000 00000 00004 27617
9	-0.00000 00000 00000 00001 44806
10	0.00000 00000 00000 00000 44795
11	-0.00000 00000 00000 00000 12709
12	0.00000 00000 00000 00000 03376
13	-0.00000 00000 00000 00000 00805
14	0.00000 00000 00000 00000 00181
15	-0.00000 00000 00000 00000 00030
16	0.00000 00000 00000 00000 00007
17	-0.00000 00000 00000 00000 00001

r	$C_{r,n}$, n = 17
0	-0.00000 00000 00000 00002 65307
1	0.00000 00000 00000 00004 91759
2	-0.00000 00000 00000 00003 92744
3	0.00000 00000 00000 00002 70777
4	-0.00000 00000 00000 00001 63029
5	0.00000 00000 00000 00000 84360
6	-0.00000 00000 00000 00000 40504
7	0.00000 00000 00000 00000 17069
8	-0.00000 00000 00000 00000 06472
9	0.00000 00000 00000 00000 02228
10	-0.00000 00000 00000 00000 00701
11	0.00000 00000 00000 00000 00202
12	-0.00000 00000 00000 00000 00054
13	0.00000 00000 00000 00000 00013
14	-0.00000 00000 00000 00000 00003
15	0.00000 00000 00000 00000 00001

r	$C_{r,n}$, n = 18
0	0.00000 00000 00000 00000 03320
1	-0.00000 00000 00000 00000 06173
2	0.00000 00000 00000 00000 04941
3	-0.00000 00000 00000 00000 03433
4	0.00000 00000 00000 00000 02083
5	-0.00000 00000 00000 00000 01114
6	0.00000 00000 00000 00000 00528
7	-0.00000 00000 00000 00000 00225
8	0.00000 00000 00000 00000 00087
9	-0.00000 00000 00000 00000 00030
10	0.00000 00000 00000 00000 00010
11	-0.00000 00000 00000 00000 00003
12	0.00000 00000 00000 00000 00001

r	$C_{r,n}$, n = 19
0	-0.00000 00000 00000 00000 00037
1	0.00000 00000 00000 00000 00069
2	-0.00000 00000 00000 00000 00056
3	0.00000 00000 00000 00000 00039
4	-0.00000 00000 00000 00000 00024
5	0.00000 00000 00000 00000 00013
6	-0.00000 00000 00000 00000 00006
7	0.00000 00000 00000 00000 00003
8	-0.00000 00000 00000 00000 00001

r	$C_{r,n}$, n = 20
0	0.00000 00000 00000 00000 00000
1	-0.00000 00000 00000 00000 00001
2	0.00000 00000 00000 00000 00001

TABLE 7

Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/6) \quad , \quad 0 < z < 8.0$$

$$A_k(\nu) = \sum_{r=0}^{\infty} B_{r,k} T_r^*\left(\frac{\nu-4}{4}\right) \quad , \quad 4 \le \nu \le 8.0$$

$B_{r,k}$, $k = 0$

r					
0	0.999816	463781	280872	208174	179446	69
1	-0.066728	085567	881465	846893	191335	91
2	0.000819	659064	439098	393101	208068	19
3	-0.000010	544089	444083	894828	736416	97
4	0.000000	570643	168262	889958	883138	05
5	-0.000001	555787	572883	840648	387997	63
6	0.000000	463004	288958	143733	485961	67
7	-0.000000	008787	623376	548767	284452	16
8	0.000000	014618	994888	133984	388688	64
9	-0.000000	000893	446685	178897	186436	94
10	-0.000000	000025	899988	466658	782933	48
11	0.000000	000010	977201	541187	644484	14
12	-0.000000	000002	123956	746643	111927	10
13	0.000000	000000	290665	572119	869829	28
14	-0.000000	000000	049992	511987	723115	37
15	-0.000000	000000	005582	644604	338669	08
16	0.000000	000000	000174	348864	277263	01
17	-0.000000	000000	000028	311468	664484	99
18	0.000000	000000	000002	465993	139082	44
19	0.000000	000000	000000	012784	078899	29
20	-0.000000	000000	000000	000864	258277	32
21	0.000000	000000	000000	013078	989328	80
22	-0.000000	000000	000000	001744	429885	08
23	0.000000	000000	000000	000114	116127	29
24	-0.000000	000000	000000	000000	047664	57
25	-0.000000	000000	000000	000000	298174	07
26	0.000000	000000	000000	000000	041284	96
27	-0.000000	000000	000000	000000	006495	63
28	0.000000	000000	000000	000000	000738	06
29	-0.000000	000000	000000	000000	000088	78
30	-0.000000	000000	000000	000000	000008	31
31	0.000000	000000	000000	000000	000000	04
32	-0.000000	000000	000000	000000	000000	01

$B_{r,k}$, $k = 1$

r					
0	-0.000023	466788	878882	563357	281840	81
1	0.000041	573282	186454	826482	184648	17
2	-0.000029	872722	881854	798818	587548	45
3	0.000016	246254	184254	875463	998165	91
4	-0.000007	349[illegible]	[illegible]776998	775867	885999	11
5	0.000002	[illegible]	861764	884778	563887	15
6	-0.000000	835847	448888	328134	612767	53
7	0.000000	287868	888831	184518	185898	80
8	-0.000000	044113	788784	887888	758648	93
9	0.000000	885346	881158	944865	715688	87
10	-0.000000	088418	384818	788215	988888	87
11	-0.000000	888815	784367	886897	694869	88
12	0.000000	888885	847872	288458	724698	88
13	-0.000000	888881	218848	488887	111338	88
14	0.000000	888884	188361	139451	322882	81
15	-0.000000	888888	817238	881868	177118	81
16	-0.000000	888888	881112	188118	558887	58
17	0.000000	888888	888888	428685	538815	68
18	-0.000000	888888	888818	738888	888238	17
19	0.000000	888888	888881	885982	588887	85
20	-0.000000	888888	888888	177267	888888	18
21	0.000000	888888	888888	888876	881888	88
22	0.000000	888888	888888	888182	177882	82
23	-0.000000	888888	888888	888815	188585	88
24	0.000000	888888	888888	788885	588888	21
25	-0.000000	888888	888888	888888	881882	77
26	0.000000	888888	888888	888888	888838	26
27	0.000000	888888	888888	888888	888888	91
28	-0.000000	888888	888888	888888	888888	41
29	0.000000	888888	888888	888888	888888	88
30	-0.000000	888888	888888	888888	888888	95
31	0.000000	888888	888888	888888	888888	17

$B_{r,k}$, $k =$

r					
0	0.000000	810588	184888	888828	838888	88
1	-0.000015	848718	852715	328778	218588	68
2	0.000010	818888	788888	888328	888881	15
3	-0.000006	128388	868388	878881	318885	12
4	0.000003	858188	384781	788138	828158	88
5	-0.000001	228938	788588	888152	888885	85
6	0.000000	818228	818758	818588	888888	87
7	-0.000000	118778	888718	838828	558832	81
8	0.000000	828858	888118	588885	518883	78
9	-0.000000	885888	188885	822881	527582	11
10	0.000000	888881	881848	858818	582728	77
11	-0.000000	888888	738981	288888	178178	71
12	0.000000	888888	888184	888858	188838	13
13	0.000000	888888	188478	158883	128883	28
14	-0.000000	888888	888881	288888	815888	88
15	0.000000	888888	817888	588888	558858	83
16	-0.000000	888888	888288	878888	878858	25
17	0.000000	888888	888825	888885	818828	28
18	-0.000000	888888	888888	888881	818788	88
19	-0.000000	888888	888888	258858	888188	18
20	0.000000	888888	888888	878833	788881	13
21	-0.000000	888888	888888	818282	888858	87
22	0.000000	888888	888888	888185	818782	19
23	-0.000000	888888	888888	888811	818718	77
24	0.000000	888888	888888	888888	278278	72
25	0.000000	888888	888888	888888	188885	88
26	-0.000000	888888	888888	888888	828782	81
27	0.000000	888888	888888	888888	888781	52
28	-0.000000	888888	888888	888888	888828	88
29	0.000000	888888	888888	888888	888881	51
30	0.000000	888888	888888	888888	888888	11
31	-0.000000	888888	888888	888888	888888	82

$B_{r,k}$, $k =$

r					
0	-0.000001	787888	112881	188887	888388	81
1	0.000003	148138	128582	188187	188888	47
2	-0.000002	111178	121121	518488	188881	14
3	0.000001	388885	858888	858878	828882	18
4	-0.000000	288728	288888	888888	128828	11
5	0.000000	182338	588588	788888	188888	88
6	-0.000000	111888	288821	188888	288888	87
7	0.000000	838882	828582	828882	788888	1
8	-0.000000	888588	388888	588838	588888	88
9	0.000000	888888	888888	888888	888888	88
10	-0.000000	888888	818888	178888	888888	88
11	0.000000	888887	818888	888888	888888	88
12	-0.000000	888881	111181	871882	888888	88
13	0.000000	888888	118888	888888	888888	88
14	-0.000000	888888	888881	888888	888888	88
15	0.000000	888888	888888	888888	888888	88
16	-0.000000	888888	888881	888888	888888	88
17	0.000000	888888	888881	888888	888888	88
18	-0.000000	888888	888888	888888	888888	88
19	0.000000	888888	888888	888888	888888	88
20	-0.000000	888888	888888	888888	888888	88
21	0.000000	888888	888888	888888	888888	88
22	-0.000000	888888	888888	888888	888888	88
23	0.000000	888888	888888	888888	888888	88
24	-0.000000	888888	888888	888888	888888	88
25	0.000000	888888	888888	888888	888888	88
26	-0.000000	888888	888888	888888	888888	88
27	0.000000	888888	888888	888888	888888	88
28	-0.000000	888888	888888	888888	888888	88
29	0.000000	888888	888888	888888	888888	88
30	-0.000000	888888	888888	888888	888888	88
31	0.000000	888888	888888	888888	888888	88

TABLE F (Continued)

Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/6) \quad , \quad 0 \le z \le 6 \quad ,$$

$$A_k(\nu) = \sum_{r=0}^{8} D_{r,k} T_r^*\left(\frac{\nu-4}{4}\right) \quad , \quad 4 \le \nu \le 8 \quad .$$

$D_{r,k}$, $k = 4$

r					
0	0.00000	23581	04070	47305	00015
1	-0.00000	47275	01744	40970	00040
2	0.00000	31691	17740	70422	70005
3	-0.00000	10011	20577	74030	47374
4	0.00000	10374	10505	01407	02303
5	-0.00000	00435	25505	30410	55035
6	0.00000	01707	13705	40007	45361
7	-0.00000	00000	15372	01430	65271
8	0.00000	00174	01051	40000	32000
9	-0.00000	00005	00000	10002	01002
10	0.00000	00010	55401	35000	00007
11	-0.00000	00002	14273	33475	11027
12	0.00000	00000	30701	07506	00003
13	-0.00000	00000	05901	27471	25023
14	0.00000	00000	00763	05500	00707
15	-0.00000	00000	00075	05177	70000
16	0.00000	00000	00003	00313	07056
17	0.00000	00000	00000	50000	00700
18	-0.00000	00000	00000	21501	05120
19	0.00000	00000	00000	04044	30104
20	-0.00000	00000	00000	00500	00200
21	0.00000	00000	00000	00062	00070
22	-0.00000	00000	00000	00005	10040
23	0.00000	00000	00000	00000	20350
24	0.00000	00000	00000	00004	02555
25	-0.00000	00000	00000	00000	00737
26	0.00000	00000	00000	00000	00100
27	-0.00000	00000	00000	00000	00011
28	0.00000	00000	00000	00000	00000
29	-0.00000	00000	00000	00000	00000

$D_{r,k}$, $k = 5$

r					
0	-0.00000	07234	34002	01554	00000
1	0.00000	00000	27007	71277	32014
2	-0.00000	03074	05736	01550	36700
3	0.00000	01950	30000	07000	00001
4	-0.00000	01049	03055	21544	53100
5	0.00000	00000	35575	36071	00103
6	-0.00000	00104	03003	33670	03000
7	0.00000	00000	50050	34214	00005
8	-0.00000	00071	34000	00001	35513
9	0.00000	00005	01052	01007	00502
10	-0.00000	00001	00354	00000	15100
11	0.00000	00000	12510	00003	20000
12	-0.00000	00000	00007	15271	03051
13	0.00000	00000	01101	20232	50000
14	-0.00000	00000	00105	07051	05003
15	0.00000	00000	00006	17267	00002
16	-0.00000	00000	00003	17736	01219
17	0.00000	00000	00000	31231	50177
18	-0.00000	00000	00000	02000	70453
19	-0.00000	00000	00000	00040	17071
20	0.00000	00000	00000	00002	11005
21	-0.00000	00000	00000	00000	51120
22	0.00000	00000	00000	00001	21002
23	-0.00000	00000	00000	00000	13050
24	0.00000	00000	00000	00000	01207
25	-0.00000	00000	00000	00000	00070
26	0.00000	00000	00000	00000	00000
27	0.00000	00000	00000	00000	00000
28	-0.00000	00000	00000	00000	00000
29	0.00000	00000	00000	00000	00000

$D_{r,k}$, $k = 6$

r					
0	0.00000	00157	03040	72454	00026
1	-0.00000	00200	03025	44470	07405
2	0.00000	00219	22403	50000	00302
3	-0.00000	00161	01937	07214	30274
4	0.00000	00077	04711	03000	70170
5	-0.00000	00030	00000	07333	02007
6	0.00000	00010	13400	00034	07137
7	-0.00000	00005	00117	01030	20200
8	0.00000	00001	01703	32100	07500
9	-0.00000	00000	02775	04304	00002
10	0.00000	00000	13700	07200	00751
11	-0.00000	00000	01250	10100	00027
12	0.00000	00000	00000	30000	00276
13	-0.00000	00000	00134	05005	00055
14	0.00000	00000	00026	11427	00000
15	-0.00000	00000	00003	00227	00105
16	0.00000	00000	00000	50537	00000
17	-0.00000	00000	00000	07301	07301
18	0.00000	00000	00000	00000	27000
19	-0.00000	00000	00000	00002	00770
20	0.00000	00000	00000	00000	03070
21	-0.00000	00000	00000	00000	10000
22	-0.00000	00000	00000	00000	00057
23	0.00000	00000	00000	00000	01111
24	-0.00000	00000	00000	00000	00107
25	0.00000	00000	00000	00000	00019
26	-0.00000	00000	00000	00000	00001
27	0.00000	00000	00000	00000	00000
28	-0.00000	00000	00000	00000	00000

$D_{r,k}$, $k = 7$

r					
0	-0.00000	00000	57301	10700	00705
1	0.00000	00015	01400	00720	07021
2	-0.00000	00012	00274	01400	50270
3	0.00000	00007	07000	00012	01270
4	-0.00000	00004	01001	20107	15111
5	0.00000	00002	10707	10055	07000
6	-0.00000	00000	01401	10000	11701
7	0.00000	00000	30010	30017	50213
8	-0.00000	00000	11552	01550	15174
9	0.00000	00000	03402	20001	30019
10	-0.00000	00000	00000	03507	33705
11	0.00000	00000	00234	75003	07271
12	-0.00000	00000	00002	00710	77700
13	0.00000	00000	00010	00077	70100
14	-0.00000	00000	00002	12001	05027
15	0.00000	00000	00000	30134	00010
16	-0.00000	00000	00000	05707	07553
17	0.00000	00000	00000	00050	13701
18	-0.00000	00000	00000	00116	21155
19	0.00000	00000	00000	00013	00500
20	-0.00000	00000	00000	00001	02303
21	0.00000	00000	00000	00000	10537
22	-0.00000	00000	00000	00000	01131
23	0.00000	00000	00000	00000	00057
24	0.00000	00000	00000	00000	00001
25	-0.00000	00000	00000	00000	00000
26	0.00000	00000	00000	00000	00000
27	-0.00000	00000	00000	00000	00000

TABLE 2 (Continued)

"Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{\infty} A_k(\nu) T_{2k}(z/8) \quad , \quad 0 < z < 8$$

$$A_k(\nu) = \sum_{r=0}^{\infty} a_{r,k} T_r\left(\frac{2\nu^2}{\nu}\right) \quad , \quad 4 \le \nu < \infty$$

$a_{r,k}$, $k = 0$

r					
0	0.99999	99999	99997	99959	99917	28
1	-0.49999	99999	97997	99827	99483	84
2	0.09999	99999	92999	79979	99459	69
3	-0.00999	99999	39997	37823	38799	34
4	0.00099	99999	99999	99997	99871	38
5	-0.00009	99999	99999	99999	67309	51
6	0.00000	99999	99991	99921	10478	25
7	-0.00000	99999	99999	99997	61812	58
8	0.00000	99999	99999	99979	99919	73
9	-0.00000	99999	99999	99977	91637	90
10	0.00000	99999	99999	99999	97961	38
11	-0.00000	99999	99999	99997	93759	36
12	0.00000	99999	99999	99991	73888	93
13	-0.00000	99999	99999	99999	13881	69
14	0.00000	99999	99999	99999	91488	35
15	-0.00000	99999	99999	99999	68769	93
16	0.00000	99999	99999	99999	40888	28
17	-0.00000	99999	99999	99999	19999	79
18	0.00000	99999	99999	99999	87773	86
19	-0.00000	99999	99999	99999	27958	19
20	0.00000	99999	99999	99999	15981	10
21	-0.00000	99999	99999	99999	81958	46
22	0.00000	99999	99999	99999	88181	67
23	-0.00000	99999	99999	99999	44332	86
24	0.00000	99999	99999	99999	19981	51
25	-0.00000	99999	99999	99999	88888	89

$a_{r,k}$, $k = 1$

r					
0	-0.00000	00000	01399	97666	73866	77
1	0.00000	00000	07342	47916	98789	77
2	-0.00000	00000	91871	96586	73737	73
3	0.00000	00000	81742	38269	19489	73
4	-0.00000	00000	98721	51859	29988	62
5	0.00000	00000	99366	98815	96826	96
6	-0.00000	00000	99161	88976	51916	31
7	0.00000	00000	99993	83822	97282	26
8	-0.00000	00000	99992	98653	38189	90
9	0.00000	00000	99997	28838	13361	96
10	-0.00000	00000	99992	99726	57767	76
11	0.00000	00000	99999	55823	67661	57
12	-0.00000	00000	99999	13611	99866	27
13	0.00000	00000	99999	93869	15887	87
14	-0.00000	00000	99999	88871	67815	56
15	0.00000	00000	99999	98119	48769	69
16	-0.00000	00000	99999	99871	66829	61
17	0.00000	00000	99999	99991	98168	26
18	-0.00000	00000	99999	99999	59135	76
19	0.00000	00000	99999	99999	88817	36
20	-0.00000	00000	99999	99999	91998	37
21	0.00000	00000	99999	99999	98138	76
22	-0.00000	00000	99999	99999	99819	84
23	0.00000	00000	99999	99999	99991	76
24	-0.00000	00000	99999	99999	99999	17
25	0.00000	00000	99999	99999	99999	82

$a_{r,k}$, $k = 2$

r					
0	0.00000	00001	44434	13619	77844	35
1	-0.00000	00001	98978	39736	73961	24
2	0.00000	00000	99999	36611	68787	87
3	-0.00000	00000	99997	37329	76796	81
4	0.00000	00000	99999	99971	98977	78
5	-0.00000	00000	99999	99911	27475	23
6	0.00000	00000	99999	99999	94634	39
7	-0.00000	00000	99999	99992	82796	28
8	0.00000	00000	99999	99999	73189	71
9	-0.00000	00000	99999	99999	29847	76
10	0.00000	00000	99999	99999	97895	21
11	-0.00000	00000	99999	99999	31978	98
12	0.00000	00000	99999	99999	97319	67
13	-0.00000	00000	99999	99999	89111	28
14	0.00000	00000	99999	99999	99823	81
15	-0.00000	00000	99999	99999	98884	73
16	0.00000	00000	99999	99999	99999	47
17	-0.00000	00000	99999	99999	99999	15
18	0.00000	00000	99999	99999	99999	87
19	-0.00000	00000	99999	99999	99999	66
20	0.00000	00000	99999	99999	99999	99
21	-0.00000	00000	99999	99999	99999	22
22	0.00000	00000	99999	99999	99999	91
23	-0.00000	00000	99999	99999	99999	11
24	0.00000	00000	99999	99999	99999	81

$a_{r,k}$, $k = 11$

r					
0	-0.00000	00000	00000	00000	13869	56
1	0.00000	00000	00000	00001	76817	23
2	-0.00000	00000	00000	00001	38228	78
3	0.00000	00000	00000	00000	00107	48
4	-0.00000	00000	00000	00000	99999	78
5	0.00000	00000	00000	00000	29817	63
6	-0.00000	00000	00000	00000	13781	67
7	0.00000	00000	00000	00000	99187	17
8	-0.00000	00000	00000	00000	91998	99
9	0.00000	00000	00000	00000	99999	17
10	-0.00000	00000	00000	00000	99798	68
11	0.00000	00000	00000	00000	99999	71
12	-0.00000	00000	00000	00000	99916	11
13	0.00000	00000	00000	00000	99999	37
14	-0.00000	00000	00000	00000	99999	87
15	0.00000	00000	00000	00000	99999	78
16	-0.00000	00000	00000	00000	99999	51
17	0.00000	00000	00000	00000	99999	88
18	-0.00000	00000	00000	00000	99999	97
19	0.00000	00000	00000	00000	99999	87
20	-0.00000	00000	00000	00000	99999	87
21	0.00000	00000	00000	00000	99999	24
22	-0.00000	00000	00000	00000	99999	96

TABLE 9 (Concluded)

Coefficients in the Expansion of

$$J_\nu(z) = z^\nu \sum_{k=0}^{n} A_k(\nu) T_{2k}(z/8) \ , \quad 0 \le z \le 8$$

$$A_k(\nu) = \sum_{r=0}^{n} B_{r,k} T_r^*\left(\frac{\nu-4}{4}\right) \ , \quad 4 \le \nu \le 8$$

r	$B_{r,k}$, k = 12	$B_{r,k}$, k = 13
0	0.00000 00000 00000 07810 42819 07	-0.00000 00000 00000 00037 02616 29
1	-0.00000 00000 00000 03721 11494 01	0.00000 00000 00000 00048 02927 79
2	0.00000 00000 00000 02998 05433 64	-0.00000 00000 00000 00044 76108 77
3	-0.00000 00000 00000 02027 10973 36	0.00000 00000 00000 00037 70017 64
4	0.00000 00000 00000 01200 00024 05	-0.00000 00000 00000 00022 75209 97
5	-0.00000 00000 00000 00634 11320 07	0.00000 00000 00000 00012 05146 94
6	0.00000 00000 00000 00294 04051 39	-0.00000 00000 00000 00005 04453 07
7	-0.00000 00000 00000 00122 10000 15	0.00000 00000 00000 00002 30231 99
8	0.00000 00000 00000 00048 05254 31	-0.00000 00000 00000 00000 00336 24
9	-0.00000 00000 00000 00015 40063 67	0.00000 00000 00000 00000 31007 79
10	0.00000 00000 00000 00004 78371 54	-0.00000 00000 00000 00000 00779 11
11	-0.00000 00000 00000 00001 38607 40	0.00000 00000 00000 00000 02024 60
12	0.00000 00000 00000 00000 38495 98	-0.00000 00000 00000 00000 00753 04
13	-0.00000 00000 00000 00000 00000 95	0.00000 00000 00000 00000 00184 13
14	0.00000 00000 00000 00000 01036 03	-0.00000 00000 00000 00000 00042 81
15	-0.00000 00000 00000 00000 00404 96	0.00000 00000 00000 00000 00009 20
16	0.00000 00000 00000 00000 00000 00	-0.00000 00000 00000 00000 00001 05
17	-0.00000 00000 00000 00000 00016 75	0.00000 00000 00000 00000 00000 35
18	0.00000 00000 00000 00000 00002 54	-0.00000 00000 00000 00000 00000 06
19	-0.00000 00000 00000 00000 00000 42	0.00000 00000 00000 00000 00000 01
20	0.00000 00000 00000 00000 00000 06	
21	-0.00000 00000 00000 00000 00000 01	

r	$B_{r,k}$, k = 14	$B_{r,k}$, k = 15
0	0.00000 00000 00000 00000 00478 68	-0.00000 00000 00000 00000 00044 95
1	-0.00000 00000 00000 00001 10755 39	0.00000 00000 00000 00000 01577 45
2	0.00000 00000 00000 00000 00877 14	-0.00000 00000 00000 00000 01243 36
3	-0.00000 00000 00000 00000 01585 07	0.00000 00000 00000 00000 00087 07
4	0.00000 00000 00000 00000 37375 04	-0.00000 00000 00000 00000 00041 04
5	-0.00000 00000 00000 00000 19007 82	0.00000 00000 00000 00000 00291 90
6	0.00000 00000 00000 00000 00499 10	-0.00000 00000 00000 00000 00140 11
7	-0.00000 00000 00000 00000 00044 36	0.00000 00000 00000 00000 00008 36
8	0.00000 00000 00000 00000 01954 24	-0.00000 00000 00000 00000 00023 67
9	-0.00000 00000 00000 00000 00542 80	0.00000 00000 00000 00000 00000 31
10	0.00000 00000 00000 00000 00173 32	-0.00000 00000 00000 00000 00002 69
11	-0.00000 00000 00000 00000 00050 00	0.00000 00000 00000 00000 00000 00
12	0.00000 00000 00000 00000 00013 04	-0.00000 00000 00000 00000 00000 22
13	-0.00000 00000 00000 00000 00001 67	0.00000 00000 00000 00000 00000 04
14	0.00000 00000 00000 00000 00000 01	-0.00000 00000 00000 00000 00000 01
15	-0.00000 00000 00000 00000 00000 18	
16	0.00000 00000 00000 00000 00000 04	
17	-0.00000 00000 00000 00000 00000 01	

r	$B_{r,k}$, k = 16	$B_{r,k}$, k = 17
0	0.00000 00000 00000 00000 00010 74	-0.00000 00000 00000 00000 00000 17
1	-0.00000 00000 00000 00000 00019 90	0.00000 00000 00000 00000 00000 23
2	0.00000 00000 00000 00000 00014 10	-0.00000 00000 00000 00000 00000 19
3	-0.00000 00000 00000 00000 00011 30	0.00000 00000 00000 00000 00000 13
4	0.00000 00000 00000 00000 00006 96	-0.00000 00000 00000 00000 00000 08
5	-0.00000 00000 00000 00000 00003 70	0.00000 00000 00000 00000 00000 04
6	0.00000 00000 00000 00000 00001 03	-0.00000 00000 00000 00000 00000 02
7	-0.00000 00000 00000 00000 00000 00	0.00000 00000 00000 00000 00000 01
8	0.00000 00000 00000 00000 00000 31	
9	-0.00000 00000 00000 00000 00000 11	
10	0.00000 00000 00000 00000 00000 04	
11	-0.00000 00000 00000 00000 00000 01	

TABLE 3

Coefficients in the expansion of

$$J_\nu(z) + iY_\nu(z) = (2/\pi z)^{1/2} e^{i(z-\frac{1}{2}\nu\pi-\pi/4)} \sum_{k=0}^{n} a_k(\nu) P_k^*(2/z) + \varepsilon_n ,$$

$$a_k(\nu) = \sum_{r=0}^{n} \beta_{r,k} P_r^*(\nu) , \quad 0 \le \nu \le 1 ,$$

$$\beta_{r,k} = p_{r,k} + i q_{r,k} .$$

r	$p_{r,k}$, k = 0	$q_{r,k}$, k = 0
0	1.000010	0.00023
1	0.000166	0.09477
2	0.00017	0.00023
3	-0.00011	0.00002
4	-0.00001	0.00006
5	-0.00000	-0.00000
6	-0.00000	-0.00000
7	0.00000	-0.00000
8	0.00000	0.00000
9	0.00000	0.00000
10	0.00000	0.00000
11	-0.00000	0.00000
12	0.00000	0.00000
13	-0.00000	-0.00000
14	-0.00000	-0.00000
15	-0.00000	-0.00000
16	-0.00000	-0.00000
17	0.00000	-0.00000

r	$p_{r,k}$, k = 1	$q_{r,k}$, k = 1
0	0.00077	0.00027
1	0.00144	0.07700
2	0.00077	0.00027
3	-0.00014	0.00003
4	-0.00001	0.00000
5	-0.00000	-0.00000
6	-0.00000	-0.00000
7	0.00000	-0.00000
8	0.00000	0.00000
9	0.00000	0.00000
10	0.00000	0.00000
11	-0.00000	0.00000
12	-0.00000	0.00000
13	-0.00000	-0.00000
14	-0.00000	-0.00000
15	-0.00000	-0.00000
16	-0.00000	-0.00000
17	0.00000	-0.00000

r	$p_{r,k}$, k = 2	$q_{r,k}$, k = 2
0	0.00077	0.00027
1	0.00144	0.07700
2	0.00077	0.00027
3	-0.00014	0.00003
4	-0.00001	0.00000
5	-0.00000	-0.00000
6	-0.00000	-0.00000
7	0.00000	-0.00000
8	0.00000	0.00000
9	0.00000	0.00000
10	0.00000	0.00000
11	-0.00000	0.00000
12	-0.00000	0.00000
13	-0.00000	-0.00000
14	-0.00000	-0.00000
15	-0.00000	-0.00000
16	-0.00000	-0.00000
17	0.00000	-0.00000

r	$p_{r,k}$, k = 3	$q_{r,k}$, k = 3
0	0.00000	-0.00001
1	0.00000	-0.00012
2	-0.00001	-0.00001
3	-0.00001	0.00001
4	0.00000	0.00000
5	-0.00000	-0.00000
6	-0.00000	-0.00000
7	-0.00000	-0.00000
8	-0.00000	-0.00000
9	0.00000	0.00000
10	0.00000	0.00000
11	0.00000	0.00000
12	0.00000	0.00000
13	0.00000	0.00000
14	-0.00000	-0.00000
15	-0.00000	-0.00000
16	-0.00000	-0.00000
17	-0.00000	-0.00000

TABLE 5 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) + iY_\nu(z) = (2/\pi z)^{\frac{1}{2}} e^{i(z-\frac{1}{2}\nu\pi-\pi/4)} \sum_{r=0}^{\infty} a_r(\nu) P_r^0(1/z) , \quad z > 8 ,$$

$$a_r(\nu) = \sum_{s=0}^{\infty} E_{r,s} \nu^{2s}(\nu) , \quad 0 \le \nu \le 1 ,$$

$$E_{r,s} = P_{r,s} + i Q_{r,s}$$

$P_{r,s}$, $s = 0$

r					
0	-0.00000	07404	70936	27000	27964
1	-0.00001	00221	00031	17002	00324
2	-0.00000	00457	17707	07108	72001
3	0.00000	10950	54011	31530	25504
4	0.00000	00733	67023	02000	20025
5	-0.00000	00102	57157	31535	00177
6	-0.00000	00011	00102	03706	27746
7	0.00000	00000	01443	71000	00441
8	0.00000	00000	00002	00025	02703
9	0.00000	00000	00100	00013	67534
10	0.00000	00000	00000	02103	17001
11	-0.00000	00000	00000	00027	00330
12	-0.00000	00000	00000	00306	70710
13	-0.00000	00000	00000	00026	67653
14	-0.00000	00000	00000	00001	00160
15	-0.00000	00000	00000	00000	01109
16	-0.00000	00000	00000	00000	00026
17	0.00000	00000	00000	00000	00001

$Q_{r,s}$, $s = 0$

r					
0	-0.00000	17066	63120	42516	30070
1	-0.00001	00120	51005	00000	51307
2	-0.00000	19670	00402	72112	70004
3	0.00000	10771	20000	25022	00000
4	0.00000	01007	10005	00000	01704
5	-0.00000	00176	37026	02001	10701
6	-0.00000	00010	00052	00030	20574
7	-0.00000	00000	70000	00012	27012
8	-0.00000	00000	00024	70034	00070
9	0.00000	00000	00003	10710	00730
10	0.00000	00000	00000	00000	71000
11	0.00000	00000	00000	00300	02004
12	0.00000	00000	00000	01107	00000
13	0.00000	00000	00000	00004	73000
14	-0.00000	00000	00000	00000	00020
15	-0.00000	00000	00000	00000	01005
16	-0.00000	00000	00000	00000	00002
17	-0.00000	00000	00000	00000	00001

$P_{r,s}$, $s = 4$

r					
0	-0.00000	00707	10007	00134	40000
1	-0.00000	00005	75004	00000	00372
2	-0.00000	00037	25013	10152	00205
3	0.00000	00074	44070	30011	10010
4	0.00000	00008	70001	04707	02001
5	-0.00000	00017	00131	20170	10100
6	-0.00000	00001	17007	20150	00000
7	-0.00000	00000	00070	02000	01277
8	0.00000	00000	00100	00030	30112
9	0.00000	00000	00000	03000	03027
10	0.00000	00000	00002	13100	00004
11	0.00000	00000	00000	01000	11300
12	-0.00000	00000	00000	00023	01500
13	-0.00000	00000	00000	00000	15000
14	-0.00000	00000	00000	00000	30002
15	-0.00000	00000	00000	00000	00503
16	-0.00000	00000	00000	00000	00015

$Q_{r,s}$, $s = 4$

r					
0	0.00000	00007	02767	00030	71100
1	0.00000	10701	00707	73043	00010
2	0.00000	00544	10043	02203	20000
3	-0.00000	01115	44020	05700	75304
4	-0.00000	00002	30004	04101	00401
5	0.00000	00022	73240	03705	70047
6	0.00000	00001	31000	13004	37011
7	-0.00000	00000	12077	02401	04703
8	-0.00000	00000	00700	02000	04705
9	0.00000	00000	00001	24313	01002
10	0.00000	00000	00000	00133	42033
11	0.00000	00000	00000	07205	20072
12	0.00000	00000	00000	00326	00703
13	0.00000	00000	00000	00003	04310
14	0.00000	00000	00000	00000	07372
15	-0.00000	00000	00000	00000	00005
16	-0.00000	00000	00000	00000	00025
17	-0.00000	00000	00000	00000	00001

$P_{r,s}$, $s = 8$

r					
0	0.00000	00054	50700	02004	10010
1	0.00000	01124	31207	44301	73000
2	0.00000	00050	52125	54014	44005
3	-0.00000	00119	01452	24004	04070
4	-0.00000	00005	07044	10000	20004
5	0.00000	00002	63704	57540	03002
6	0.00000	00004	13410	15052	20300
7	-0.00000	00000	01003	01400	05330
8	-0.00000	00000	00000	04317	02074
9	0.00000	00000	00002	50401	02777
10	0.00000	00000	00000	10710	42047
11	0.00000	00000	00000	00007	47020
12	0.00000	00000	00000	00027	00004
13	-0.00000	00000	00000	00001	20004
14	-0.00000	00000	00000	00000	00047
15	-0.00000	00000	00000	00000	00202
16	-0.00000	00000	00000	00000	00004

$Q_{r,s}$, $s = 8$

r					
0	0.00000	00030	67601	74111	14077
1	0.00000	00301	00711	21101	44302
2	0.00000	00027	50005	71015	07452
3	-0.00000	00034	20030	05477	31425
4	-0.00000	00003	02320	07004	01115
5	0.00000	00000	43500	20101	12304
6	0.00000	00000	05316	53514	04315
7	0.00000	00000	00700	04443	72212
8	-0.00000	00000	00010	40003	47027
9	-0.00000	00000	00004	53317	57015
10	-0.00000	00000	00000	10007	77270
11	0.00000	00000	00000	00000	32044
12	0.00000	00000	00000	00004	01003
13	0.00000	00000	00000	00001	[illegible]
14	0.00000	00000	00000	00000	04023
15	-0.00000	00000	00000	00000	00000
16	-0.00000	00000	00000	00000	00004

TABLE 4 (Continued)

Coefficients in the Expansion of

$$J_\nu(x) + iY_\nu(x) = (2/\pi x)^{\frac{1}{2}} e^{i(x-\frac{1}{2}\nu\pi-\pi/4)} \sum_{r=0}^{\infty} B_r(\nu) T_r^8(8/x) , \quad x \geq 1 ,$$

$$B_r(\nu) = \sum_{s=0}^{\infty} B_{r,s} T_s^*(\nu) , \quad 0 \leq \nu \leq 1 ,$$

$$B_{r,s} = P_{r,s} + i Q_{r,s}$$

$P_{r,s}$

0	0.00000	00000	64355	17931	60075
1	-0.00000	00015	26817	36150	06505
2	0.00000	00007	68173	37016	00361
3	0.00000	00002	07740	61290	20751
4	-0.00000	00000	81215	58302	60734
5	-0.00000	00000	07702	67250	00077
6	-0.00000	00000	00001	06550	20707
7	0.00000	00000	00116	43644	13507
8	0.00000	00000	00007	61572	00025
9	-0.00000	00000	00000	74057	00305
10	-0.00000	00000	00000	07544	13050
11	0.00000	00000	00000	00163	00004
12	0.00000	00000	00000	00007	02700
13	0.00000	00000	00000	00000	07412
14	-0.00000	00000	00000	00000	00165
15	-0.00000	00000	00000	00000	00002
16	-0.00000	00000	00000	00000	00007

$Q_{r,s}$

0	-0.00000	00000	54312	77401	72201
1	-0.00000	00110	00010	21720	20301
2	-0.00000	00004	54170	54017	16030
3	0.00000	00012	00205	10310	32320
4	0.00000	00000	54725	00010	61004
5	-0.00000	00000	27230	37744	77701
6	-0.00000	00000	01300	00460	64010
7	0.00000	00000	00219	54450	02060
8	0.00000	00000	00011	20003	69071
9	-0.00000	00000	00000	64322	00250
10	-0.00000	00000	00000	02004	13151
11	-0.00000	00000	00000	00004	65002
12	-0.00000	00000	00000	00001	31130
13	0.00000	00000	00000	00000	31737
14	0.00000	00000	00000	00000	01231
15	0.00000	00000	00000	00000	00009

$P_{r,s}$

0	-0.00000	00000	57561	62515	70010
1	-0.00000	00011	61161	64510	70616
2	-0.00000	00005	64000	20073	60760
3	0.00000	00001	73111	00124	32723
4	0.00000	00000	05404	62000	72201
5	-0.00000	00000	02025	67214	10344
6	-0.00000	00000	00130	57243	50227
7	0.00000	00000	00021	23000	25021
8	0.00000	00000	00001	13270	25414
9	-0.00000	00000	00000	03000	00117
10	-0.00000	00000	00000	00310	00205
11	-0.00000	00000	00000	00015	67041
12	-0.00000	00000	00000	00000	15007
13	0.00000	00000	00000	00000	00475
14	0.00000	00000	00000	00000	00104
15	-0.00000	00000	00000	00000	00007

$Q_{r,s}$

0	0.00000	00000	21535	10473	97401
1	0.00000	00000	11706	10000	01115
2	0.00000	00000	10024	00320	00055
3	-0.00000	00000	02666	57717	33306
4	-0.00000	00000	07417	05300	10002
5	0.00000	00000	02530	70417	70270
6	0.00000	00000	00071	50566	00700
7	-0.00000	00000	00027	65012	56334
8	-0.00000	00000	00000	05000	00526
9	0.00000	00000	00000	13017	57657
10	0.00000	00000	00000	00007	72000
11	-0.00000	00000	00000	00020	36501
12	-0.00000	00000	00000	00001	23104
13	-0.00000	00000	00000	00000	00107
14	0.00000	00000	00000	00000	00004
15	0.00000	00000	00000	00000	00001

$P_{r,s}$

0	0.00000	00000	05191	00079	00076
1	0.00000	00001	00002	15174	00700
2	0.00000	00000	00002	00000	13771
3	-0.00000	00000	00770	70103	37601
4	-0.00000	00000	00073	01222	37000
5	0.00000	00000	00004	30707	00000
6	0.00000	00000	00015	00076	31003
7	-0.00000	00000	00005	00053	03003
8	-0.00000	00000	00000	17007	30303
9	0.00000	00000	00000	02370	20001
10	0.00000	00000	00000	00000	20700
11	-0.00000	00000	00000	00000	53701
12	-0.00000	00000	00000	00000	20020
13	-0.00000	00000	00000	00000	00005
14	0.00000	00000	00000	00000	00011
15	0.00000	00000	00000	00000	00001

$Q_{r,s}$

0	0.00000	00000	04151	43110	07500
1	0.00000	00000	75000	01100	07000
2	0.00000	00000	01701	70010	01170
3	-0.00000	00000	07003	10400	00705
4	-0.00000	00000	00030	37605	07521
5	0.00000	00000	00104	00700	13000
6	0.00000	00000	00010	30031	00000
7	-0.00000	00000	00000	05737	01307
8	-0.00000	00000	00000	00000	20000
9	-0.00000	00000	00000	00327	00000
10	0.00000	00000	00000	00014	00730
11	0.00000	00000	00000	00001	07000
12	0.00000	00000	00000	00000	07211
13	-0.00000	00000	00000	00000	01012
14	-0.00000	00000	00000	00000	00017
15	0.00000	00000	00000	00000	00001

TABLE 4 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) + iY_\nu(z) = (2/\pi z)^{\frac{1}{2}} e^{i(z-\frac{1}{2}\nu\pi-\pi/4)} \sum_{k=0}^{\infty} B_k(\nu) T_k^*(10/z) , \quad z \geq 1 ,$$

$$B_\nu(\nu) = \sum_{k=0}^{\infty} B_{\nu,k} T_k^*(\nu) , \quad 0 \leq \nu \leq 1 ,$$

$$B_{\nu,k} = P_{\nu,k} + i\, Q_{\nu,k}$$

0	0.00000	00000	94365	17911	00075
1	-0.00000	00015	76417	16150	06905
2	0.00000	00001	68173	37016	09351
3	0.00000	00002	07740	61200	20791
4	-0.00000	00000	01215	50302	40736
5	-0.00000	00000	07702	67250	00077
6	-0.00000	00000	00061	00550	20707
7	0.00000	00000	00116	43044	13567
8	0.00000	00000	00007	61572	00025
9	-0.00000	00000	00000	76057	00309
10	-0.00000	00000	00000	07544	13050
11	0.00000	00000	00000	00103	00004
12	0.00000	00000	00000	00007	02700
13	0.00000	00000	00000	00000	07412
14	-0.00000	00000	00000	00000	00105
15	-0.00000	00000	00000	00000	00002
16	-0.00000	00000	00000	00000	00007

0	-0.00000	00000	94317	77491	77203
1	-0.00000	00118	88010	21170	20301
2	-0.00000	00004	94170	44017	14030
3	0.00000	00012	00205	10314	32320
4	0.00000	00000	54725	00014	61004
5	-0.00000	00000	27230	37704	77791
6	-0.00000	00000	01390	00660	64010
7	0.00000	00000	00219	54450	42050
8	0.00000	00000	00011	20003	69073
9	-0.00000	00000	00000	66122	90259
10	-0.00000	00000	00000	02004	13151
11	-0.00000	00000	00000	00004	05002
12	-0.00000	00000	00000	00001	31130
13	0.00000	00000	00000	00000	31237
14	0.00000	00000	00000	00000	01231
15	0.00000	00000	00000	00000	00009

0	-0.00000	00000	47041	42515	70010
1	-0.00000	00011	61141	64518	70016
2	-0.00000	00000	44000	28873	40760
3	0.00000	00001	23111	90124	32723
4	0.00000	00000	05404	42000	72703
5	-0.00000	00000	02025	47214	10344
6	-0.00000	00000	00130	57243	50221
7	0.00000	00000	00021	23400	25021
8	0.00000	00000	00001	13270	25414
9	-0.00000	00000	00000	03000	00117
10	-0.00000	00000	00000	00310	00705
11	-0.00000	00000	00000	00015	47061
12	-0.00000	00000	00000	00000	15007
13	0.00000	00000	00000	00000	05475
14	0.00000	00000	00000	00000	00100
15	-0.00000	00000	00000	00000	00007

0	0.00000	00000	21535	10473	97481
1	0.00000	00000	11704	10400	01115
2	0.00000	00000	19024	00170	40955
3	-0.00000	00000	02044	57717	31304
4	-0.00000	00000	02437	05300	14007
5	0.00000	00000	02530	20612	70270
6	0.00000	00000	00071	50504	00700
7	-0.00000	00000	00027	05017	54334
8	-0.00000	00000	00004	05000	00520
9	0.00000	00000	00000	13037	51457
10	0.00000	00000	00000	00407	72040
11	-0.00000	00000	00000	00070	34511
12	-0.00000	00000	00000	00001	23100
13	-0.00000	00000	00000	00000	00107
14	0.00000	00000	00000	00000	00004
15	0.00000	00000	00000	00000	00007

0	0.00000	00000	05191	00079	00074
1	0.00000	00001	00002	15174	00700
2	0.00000	00000	00002	00000	13771
3	-0.00000	00000	10779	76103	37401
4	-0.00000	00000	00073	51272	37450
5	0.00000	00000	00004	30707	00040
6	0.00000	00000	00015	00074	31003
7	-0.00000	00000	00005	00053	03403
8	-0.00000	00000	00000	17007	30303
9	0.00000	00000	00000	02376	20001
10	0.00000	00000	00000	00000	20700
11	-0.00000	00000	00000	00004	53701
12	-0.00000	00000	00000	00000	20020
13	-0.00000	00000	00000	00000	00005
14	0.00000	00000	00000	00000	00011
15	0.00000	00000	00000	00000	00001

0	0.00000	00000	04151	43110	47549
1	0.00000	00000	75004	01114	07000
2	0.00000	00000	01701	70010	61117
3	-0.00000	00000	07003	10400	44725
4	-0.00000	00000	00439	37005	47521
5	0.00000	00000	00104	40704	13450
6	0.00000	00000	00010	30511	40000
7	-0.00000	00000	00000	05717	03307
8	-0.00000	00000	00000	00000	20504
9	-0.00000	00000	00000	00127	00450
10	0.00000	00000	00000	00014	00730
11	0.00000	00000	00000	00001	07450
12	0.00000	00000	00000	00000	07211
13	-0.00000	00000	00000	00000	01012
14	-0.00000	00000	00000	00000	00012
15	0.00000	00000	00000	00000	00001

TABLE [illegible]

Coefficients in the Expansion of

$$J_\nu(z) = (z/2)^\nu \sum_{k=0}^{\infty} a_k(\nu) z^{2k}$$

$$N_\nu(z) = \sum_{k=0}^{\infty} b_k(\nu) z^{2k} + a_\nu(z) \ln z$$

$$b_k = p_k + q_k \psi(k+1)$$

$p_k(\nu)$

0	-0.00000	00000	00000	39372	74013
1	-0.00000	00000	00010	97847	98438
2	-0.00000	00000	00099	36876	15617
3	0.00000	00000	00001	17251	11022
4	0.00000	00000	00000	04328	26901
5	-0.00000	00000	00000	02931	77744
6	-0.00000	00000	00000	00117	07610
7	0.00000	00000	00000	00026	53004
8	0.00000	00000	00000	00001	23810
9	-0.00000	00000	00000	00000	07982
10	-0.00000	00000	00000	00000	00904
11	-0.00000	00000	00000	00000	00010
12	0.00000	00000	00000	00000	00001

$p_k(\nu)$

0	0.00000	00000	00000	38007	[illegible]
1	0.00000	00000	00012	47548	[illegible]
2	0.00000	00000	00000	[illegible]	[illegible]
3	-0.00000	00000	00002	[illegible]	[illegible]
4	-0.00000	00000	00000	04134	05117
5	0.00000	00000	00000	00500	00700
6	0.00000	00000	00000	00130	07199
7	-0.00000	00000	00000	00040	50299
8	-0.00000	00000	00000	00001	00001
9	0.00000	00000	00000	00000	51002
10	0.00000	00000	00000	00000	01121
11	-0.00000	00000	00000	00000	00101
12	-0.00000	00000	00000	00000	00004

$p_k(\nu)$

0	0.00000	00000	00000	00005	00007
1	0.00000	00000	00004	27600	70700
2	0.00000	00000	00000	00037	80700
3	-0.00000	00000	00000	49100	00157
4	-0.00000	00000	00000	01124	20170
5	0.00000	00000	00000	01375	15731
6	0.00000	00000	00000	00032	56120
7	-0.00000	00000	00000	00015	97635
8	-0.00000	00000	00000	00000	39350
9	0.00000	00000	00000	00000	00103
10	0.00000	00000	00000	00000	00200
11	-0.00000	00000	00000	00000	00029
12	-0.00000	00000	00000	00000	00001

$p_k(\nu)$

0	0.00000	00000	00000	01700	51701
1	-0.00000	00000	00000	01105	[illegible]
2	0.00000	00000	00000	01722	00021
3	-0.00000	00000	00000	10565	00021
4	-0.00000	00000	00000	00171	70050
5	-0.00000	00000	00000	00171	00137
6	0.00000	00000	00000	00001	93761
7	0.00000	00000	00000	00005	98750
8	0.00000	00000	00000	00000	01190
9	-0.00000	00000	00000	00000	00202
10	-0.00000	00000	00000	00000	00042
11	0.00000	00000	00000	00000	00020

$p_k(\nu)$

0	-0.00000	00000	00000	01052	05077
1	-0.00000	00000	00000	00100	01110
2	-0.00000	00000	00000	00020	03010
3	0.00000	00000	00000	00170	10071
4	0.00000	00000	00000	00121	30092
5	-0.00000	00000	00000	00020	04009
6	-0.00000	00000	00000	00001	77400
7	0.00000	00000	00000	00001	00700
8	0.00000	00000	00000	00000	00110
9	-0.00000	00000	00000	00000	00110
10	-0.00000	00000	00000	00000	00030
11	0.00000	00000	00000	00000	00000

$p_k(\nu)$

0	-0.00000	00000	00000	01011	10710
1	-0.00000	00000	00000	[illegible]	[illegible]
2	-0.00000	00000	00000	01000	[illegible]
3	0.00000	00000	00000	00251	[illegible]
4	0.00000	00000	00000	00107	[illegible]
5	-0.00000	00000	00000	00100	[illegible]
6	-0.00000	00000	00000	00005	[illegible]
7	0.00000	00000	00000	00001	[illegible]
8	0.00000	00000	00000	00000	[illegible]
9	-0.00000	00000	00000	00000	[illegible]
10	-0.00000	00000	00000	00000	[illegible]
11	0.00000	00000	00000	00000	[illegible]

TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) + iY_\nu(z) = (2/\pi z)^{\frac{1}{2}} e^{i(z-\frac{1}{2}\nu\pi-\pi/4)} \sum_{k=0}^{n} a_k(\nu) z^{-k}(1/z) \ , \quad |\arg z| < \pi \ ,$$

$$a_k(\nu) = \sum_{r=0}^{n} E_{r,k} \nu^r(\nu) \ , \quad 0 \le \nu \le 1 \ ,$$

$$E_{r,k} = P_{r,k} + i Q_{r,k}$$

$P_{r,k}$, $k = 10$

0	-0.00000	00000	00000	00100	97589
1	0.00000	00000	00000	00005	42754
2	-0.00000	00000	00000	00000	99564
3	-0.00000	00000	00000	00163	84726
4	0.00000	00000	00000	00011	68367
5	0.00000	00000	00000	00007	09401
6	-0.00000	00000	00000	00000	88433
7	-0.00000	00000	00000	00000	15230
8	0.00000	00000	00000	00000	00726
9	0.00000	00000	00000	00000	00104
10	-0.00000	00000	00000	00000	00000
11	-0.00000	00000	00000	00000	00001

$Q_{r,k}$, $k = 10$

0	0.00000	00000	00000	00376	60767
1	0.00000	00000	00000	10076	30900
2	0.00000	00000	00000	00330	86899
3	-0.00000	00000	00000	02167	06702
4	-0.00000	00000	00000	00042	49023
5	0.00000	00000	00000	00042	09177
6	0.00000	00000	00000	00001	29431
7	-0.00000	00000	00000	00000	19429
8	-0.00000	00000	00000	00000	01509
9	0.00000	00000	00000	00000	00076
10	0.00000	00000	00000	00000	00010
11	-0.00000	00000	00000	00000	00002

$P_{r,k}$, $k = 17$

0	0.00000	00000	00000	00073	38103
1	0.00000	00000	00000	02087	44135
2	0.00000	00000	00000	00004	90131
3	-0.00000	00000	00000	00330	43383
4	-0.00000	00000	00000	00000	84009
5	0.00000	00000	00000	00009	10267
6	0.00000	00000	00000	00000	23741
7	-0.00000	00000	00000	00000	10266
8	-0.00000	00000	00000	00000	00284
9	0.00000	00000	00000	00000	00054
10	0.00000	00000	00000	00000	00007

$Q_{r,k}$, $k = 17$

0	-0.00000	00000	00000	00035	33836
1	-0.00000	00000	00000	02009	23076
2	-0.00000	00000	00000	00031	00949
3	0.00000	00000	00000	00342	71037
4	0.00000	00000	00000	00004	11059
5	-0.00000	00000	00000	00010	35615
6	-0.00000	00000	00000	00000	13282
7	0.00000	00000	00000	00000	13776
8	0.00000	00000	00000	00000	00100
9	-0.00000	00000	00000	00000	00009
10	-0.00000	00000	00000	00000	00001

$P_{r,k}$, $k = 10$

0	-0.00000	00000	00000	00010	30991
1	-0.00000	00000	00000	00027	75274
2	-0.00000	00000	00000	00014	66151
3	0.00000	00000	00000	00107	70014
4	0.00000	00000	00000	00001	86764
5	-0.00000	00000	00000	00003	13499
6	-0.00000	00000	00000	00000	05621
7	0.00000	00000	00000	00000	03022
8	0.00000	00000	00000	00000	00073
9	-0.00000	00000	00000	00000	00026
10	-0.00000	00000	00000	00000	00001

$Q_{r,k}$, $k = 10$

0	-0.00000	00000	00000	00000	44299
1	-0.00000	00000	00000	00020	33073
2	-0.00000	00000	00000	00005	72076
3	0.00000	00000	00000	00001	94469
4	0.00000	00000	00000	00000	70352
5	0.00000	00000	00000	00000	00430
6	-0.00000	00000	00000	00000	01049
7	-0.00000	00000	00000	00000	00026
8	0.00000	00000	00000	00000	00010
9	0.00000	00000	00000	00000	00004

TABLE 5 (Continued)

Coefficients in the Expansion of

$$J_\nu(z) + iY_\nu(z) \sim (2/\pi z)^{\frac{1}{2}} e^{i(z-\frac{1}{2}\nu\pi-\pi/4)} \sum_{r=0}^{\infty} R_r(\nu) T_r^2(5/z) \ , \quad z \geq 5$$

$$R_r(\nu) = \sum_{s=0}^{\infty} E_{r,s} T_r^2(\nu) \ , \quad 0 \leq \nu \leq 1 \ ,$$

$$E_{r,s} = F_{r,s} + i\,G_{r,s}$$

$F_{r,s}$, $s = 19$

r					
0	0.	00000	00000	00001	60403
1	0.	00000	00000	00164	21013
2	0.	00000	00000	00001	30407
3	-0.	00000	00000	00017	15122
4	-0.	00000	00000	00000	17410
5	0.	00000	00000	00000	02467
6	0.	00000	00000	00000	00369
7	-0.	00000	00000	00000	00713
8	-0.	00000	00000	00000	00000
9	0.	00000	00000	00000	00005

$G_{r,s}$, $s = 19$

r					
0	0.	00000	00000	00001	63001
1	0.	00000	00000	00154	99449
2	0.	00000	00000	00003	71005
3	-0.	00000	00000	00018	31993
4	-0.	00000	00000	00000	41060
5	0.	00000	00000	00000	01419
6	0.	00000	00000	00000	01202
7	-0.	00000	00000	00000	00000
8	-0.	00000	00000	00000	00019
9	0.	00000	00000	00000	00004

$F_{r,s}$, $s = 20$

r					
0	0.	00000	00000	00000	35050
1	0.	00000	00000	00007	76981
2	0.	00000	00000	00000	31002
3	-0.	00000	00000	00000	22090
4	-0.	00000	00000	00000	03049
5	-0.	00000	00000	00000	00000
6	0.	00000	00000	00000	00107
7	0.	00000	00000	00000	00017
8	-0.	00000	00000	00000	00001

$G_{r,s}$, $s = 20$

r					
0	-0.	00000	00000	00000	02927
1	-0.	00000	00000	00001	21914
2	-0.	00000	00000	00000	73151
3	0.	00000	00000	00000	00175
4	0.	00000	00000	00000	09402
5	-0.	00000	00000	00000	17044
6	-0.	00000	00000	00000	00700
7	0.	00000	00000	00000	00229
8	0.	00000	00000	00000	00004
9	-0.	00000	00000	00000	00002

$F_{r,s}$, $s = 21$

r					
0	-0.	00000	00000	00000	19029
1	-0.	00000	00000	00000	20002
2	-0.	00000	00000	00000	17513
3	0.	00000	00000	00001	07434
4	0.	00000	00000	00000	02269
5	-0.	00000	00000	00000	03050
6	-0.	00000	00000	00000	00044
7	0.	00000	00000	00000	00030
8	0.	00000	00000	00000	00001

$G_{r,s}$, $s = 21$

r					
0	0.	00000	00000	00000	07010
1	0.	00000	00000	00000	47078
2	0.	00000	00000	00000	06057
3	-0.	00000	00000	00001	00964
4	-0.	00000	00000	00000	00931
5	0.	00000	00000	00000	03107
6	0.	00000	00000	00000	00031
7	-0.	00000	00000	00000	00041

$F_{r,s}$, $s = 22$

r					
0	0.	00000	00000	00000	04705
1	0.	00000	00000	00003	15378
2	0.	00000	00000	00000	04220
3	-0.	00000	00000	00000	13049
4	-0.	00000	00000	00000	00560
5	0.	00000	00000	00000	01090
6	0.	00000	00000	00000	00017
7	-0.	00000	00000	00000	00014

$G_{r,s}$, $s = 22$

r					
0	0.	00000	00000	00000	01971
1	0.	00000	00000	00000	00000
2	0.	00000	00000	00000	01747
3	-0.	00000	00000	00000	00499
4	-0.	00000	00000	00000	00210
5	-0.	00000	00000	00000	00029
6	0.	00000	00000	00000	00000
7	0.	00000	00000	00000	00001

TABLE 1 (Continued)

Coefficients in the Expansion of

$$J_\nu(\alpha) + iY_\nu(\alpha) = (2/\pi\alpha)^{\frac{1}{2}} e^{i(\alpha - \frac{1}{2}\nu\pi - \pi/4)} \sum_{k=0}^{n} \eta_k(\nu) T_k^*(8/\alpha) , \quad \alpha \geq 8$$

$$\eta_k(\nu) = \sum_{r=0}^{n} E_{r,k} T_r^*(\nu) , \quad 0 \leq \nu \leq 1 ,$$

$$E_{r,k} = P_{r,k} + i Q_{r,k}$$

$P_{r,k}$, $k = 0$

r	
0	-0.00000 00000 00000 00000 00000
1	-0.00000 00000 00000 00000 97620
2	-0.00000 00000 00000 00000 00000
3	0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000
6	-0.00000 00000 00000 00000 00000
7	0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 0$

r	
0	-0.00000 00000 00000 00000 01171
1	-0.00000 00000 00000 00000 97656
2	-0.00000 00000 00000 00000 01074
3	0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000
6	-0.00000 00000 00000 00000 00000
7	0.00000 00000 00000 00000 00000

$P_{r,k}$, $k = 2$

r	
0	-0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	-0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 2$

r	
0	0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	-0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000
6	0.00000 00000 00000 00000 00000
7	-0.00000 00000 00000 00000 00000

$P_{r,k}$, $k = 4$

r	
0	0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	-0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 4$

r	
0	-0.00000 00000 00000 00000 00000
1	-0.00000 00000 00000 00000 00000
2	-0.00000 00000 00000 00000 00000
3	0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000

$P_{r,k}$, $k = 6$

r	
0	-0.00000 00000 00000 00000 00000
1	-0.00000 00000 00000 00000 01924
2	-0.00000 00000 00000 00000 00000
3	0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 6$

r	
0	-0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	-0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000

$P_{r,k}$, $k = 8$

r	
0	0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 8$

r	
0	-0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	-0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000

$P_{r,k}$, $k = 10$

r	
0	0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	-0.00000 00000 00000 00000 00000
3	0.00000 00000 00000 00000 00000
4	0.00000 00000 00000 00000 00000
5	-0.00000 00000 00000 00000 00000

$Q_{r,k}$, $k = 10$

r	
0	0.00000 00000 00000 00000 00000
1	0.00000 00000 00000 00000 00000
2	0.00000 00000 00000 00000 00000
3	-0.00000 00000 00000 00000 00000
4	-0.00000 00000 00000 00000 00000
5	0.00000 00000 00000 00000 00000

~~[illegible] (Continued)~~

Coefficients in the Expansion of

$$J_\nu(s) + iY_\nu(s) = (2/\pi s)^{\frac{1}{2}} e^{i(s-\frac{1}{2}\nu\pi-\pi/4)} \sum_{n=0}^{m} B_n(\nu) s^{-n}(8/s) \ , \quad s \geq 8 \ .$$

$$B_n(\nu) = \sum_{r=0}^{m} B_{r,n} \nu^r(\nu) \ , \quad 0 \leq \nu \leq 1 \ .$$

$$B_{r,n} = P_{r,n} + i Q_{r,n}$$

r $P_{r,0}$, n = 0

0 0.00000 00000 00000 00000 00000
1 -0.00000 00000 00000 00000 00036
2 0.00000 00000 00000 00000 00000
3 0.00000 00000 00000 00000 00004

r $Q_{r,0}$, n = 0

0 -0.00000 00000 00000 00000 00002
1 -0.00000 00000 00000 00000 00119
2 -0.00000 00000 00000 00000 00001
3 0.00000 00000 00000 00000 00014

r $P_{r,1}$, n = 1

0 -0.00000 00000 00000 00000 00000
1 -0.00000 00000 00000 00013 00
2 -0.00000 00000 00000 00000 00000
3 0.00000 00000 00000 00000 00002

r $Q_{r,1}$, n = 1

0 0.00000 00000 00000 00000 00000
1 0.00000 00000 00000 00000 00037
2 0.00000 00000 00000 00000 00000
3 -0.00000 00000 00000 00000 00004

r $P_{r,2}$, n = 2

0 0.00000 00000 00000 00000 00000
1 0.00000 00000 00000 00000 00005
2 0.00000 00000 00000 00000 00000
3 -0.00000 00000 00000 00000 00001

r $Q_{r,2}$, n = 2

0 -0.00000 00000 00000 00000 00000
1 -0.00000 00000 00000 00000 00005
2 -0.00000 00000 00000 00000 00000
3 0.00000 00000 00000 00000 00001

r $P_{r,3}$, n = 3

0 -0.00000 00000 00000 00000 00000
1 -0.00000 00000 00000 00000 00001

r $Q_{r,3}$, n = 3

0 -0.00000 00000 00000 00000 00000
1 -0.00000 00000 00000 00000 00001

r $P_{r,4}$, n = 4

0 0.00000 00000 00000 00000 00000
1 0.00000 00000 00000 00000 00001

r $Q_{r,4}$, n = 4

0 0.00000 00000 00000 00000 00000
1 0.00000 00000 00000 00000 00001

CORRIGENDA

Yudell L. Luke, "Miniaturized Tables of Bessel Functions. II," Math. Comp. 25(1971), 789-795 and the accompanying microfiche.

This concerns only the coefficients in the expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu)\, T_k^*(8/z), \quad z \geq 8,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k}\, T_r^*(\nu), \quad 0 \leq \nu \leq 1.$$

In the above paper, I gave the $E_{r,k}$'s, for $k = 0(1)\,20$ and all r which do not vanish, to 22 decimals. For $k = 21$, only the coefficients for $r = 0(1)5$ were given. It was stated that the coefficients are sufficiently accurate to enable the computation of $e^{-z}(2\pi z)^{\frac{1}{2}} I_\nu(z)$ to about 22 decimals. Unfortunately, as the paper stands, only about 15 decimal accuracy is possible because inadvertently the pertinent $E_{r,k}$'s for $k = 20(1)40$ were omitted. The missing coefficients are supplied here.

Actually, all the coefficients for the above expansion are given for the sake of completeness.

Yudell L. Luke

University of Missouri
Kansas City, Missouri 64110

TABLE 3

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu) \ , \ 0 \leq \nu \leq 1 \ .$$

r	$E_{r,k}$, k = 0
0	0.99601 36885 05338 32062 28
1	-0.01629 18723 83299 78423 34
2	-0.00398 00326 49983 99358 09
3	0.00005 26713 08143 15887 04
4	0.00000 62557 24366 64173 90
5	-0.00000 01006 36595 12881 05
6	-0.00000 00076 95123 50578 52
7	0.00000 00001 45705 44979 69
8	0.00000 00000 07922 57176 97
9	-0.00000 00000 00180 95644 18
10	-0.00000 00000 00007 06098 39
11	0.00000 00000 00000 20817 71
12	0.00000 00000 00000 00523 57
13	-0.00000 00000 00000 00022 42
14	-0.00000 00000 00000 00000 25
15	0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 1
0	-0.00401 40331 65105 02968 37
1	-0.01652 76673 31909 09023 13
2	-0.00400 55537 88259 83470 52
3	0.00007 15385 84267 23656 84
4	0.00000 86384 90481 04821 30
5	-0.00000 01542 85341 01459 00
6	-0.00000 00117 19182 98832 59
7	0.00000 00002 39194 71285 79
8	0.00000 00000 12901 04629 29
9	-0.00000 00000 00310 87748 11
10	-0.00000 00000 00011 97168 51
11	0.00000 00000 00000 36923 95
12	0.00000 00000 00000 00905 40
13	-0.00000 00000 00000 00040 58
14	-0.00000 00000 00000 00000 43
15	0.00000 00000 00000 00000 04

r	$E_{r,k}$, k = 2
0	-0.00002 83835 61655 55270 96
1	-0.00024 46146 55718 44712 36
2	-0.00002 61088 96649 38277 62
3	0.00001 97288 39252 72587 42
4	0.00000 22450 65467 74325 00
5	-0.00000 00673 10178 71437 91
6	-0.00000 00049 79011 31089 05
7	-0.00000 00001 31070 19933 23
8	0.00000 00000 06862 59728 70
9	-0.00000 00000 00196 71869 04
10	-0.00000 00000 00007 23110 98
11	0.00000 00000 00000 25759 45
12	0.00000 00000 00000 00578 63
13	-0.00000 00000 00000 00030 06
14	-0.00000 00000 00000 00000 25
15	0.00000 00000 00000 00000 03

r	$E_{r,k}$, k = 3
0	-0.00000 06955 69590 22719 23
1	-0.00000 93976 31721 38630 48
2	-0.00000 06108 79764 78893 94
3	0.00000 09226 98028 28737 34
4	0.00000 00651 86021 31753 10
5	-0.00000 00149 81940 84090 02
6	-0.00000 00010 16748 15913 45
7	0.00000 00000 46811 33701 05
8	0.00000 00000 02270 84124 62
9	-0.00000 00000 00091 52809 65
10	-0.00000 00000 00003 01669 80
11	0.00000 00000 00000 14155 50
12	0.00000 00000 00000 00260 67
13	-0.00000 00000 00000 00018 17
14	-0.00000 00000 00000 00000 08
15	0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 4
0	-0.00000 00381 75275 85093 63
1	-0.00000 06379 69369 39048 23
2	-0.00000 00228 37493 95620 65
3	0.00000 00677 89040 26021 41
4	0.00000 00030 09633 14738 60
5	-0.00000 00014 86421 86403 41
6	-0.00000 00000 65538 08421 68
7	0.00000 00000 10799 78762 11
8	0.00000 00000 00419 65913 59
9	-0.00000 00000 00031 61364 69
10	-0.00000 00000 00000 79674 76
11	0.00000 00000 00000 06150 19
12	0.00000 00000 00000 00069 23
13	-0.00000 00000 00000 00008 89
14	0.00000 00000 00000 00000 02
15	0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 5
0	-0.00000 00059 25360 97692 36
1	-0.00000 00688 62950 04823 26
2	0.00000 00012 68210 16741 60
3	0.00000 00076 65226 96602 42
4	0.00000 00000 88362 19310 74
5	-0.00000 00001 93188 29938 77
6	-0.00000 00000 03329 64005 74
7	0.00000 00000 01465 85438 56
8	0.00000 00000 00030 16772 40
9	-0.00000 00000 00008 47512 47
10	-0.00000 00000 00000 08430 46
11	0.00000 00000 00000 02118 99
12	-0.00000 00000 00000 00002 16
13	-0.00000 00000 00000 00003 44
14	0.00000 00000 00000 00000 05

TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu) \ , \ 0 \leq \nu \leq 1 \ .$$

r	$E_{r,k}$, k = 6	r	$E_{r,k}$, k = 7
0	-0.00000 00019 78212 12066 64	0	-0.00000 00007 30866 32786 40
1	-0.00000 00102 08031 71716 72	1	-0.00000 00014 93852 77992 09
2	0.00000 00014 85119 69317 49	2	0.00000 00006 81464 43819 15
3	0.00000 00011 76123 57379 91	3	0.00000 00001 79069 10742 40
4	-0.00000 00000 58099 59802 29	4	-0.00000 00000 32505 70263 18
5	-0.00000 00000 31921 89916 04	5	-0.00000 00000 05057 01336 39
6	0.00000 00000 00807 24683 92	6	0.00000 00000 00554 50949 13
7	0.00000 00000 00349 86250 02	7	0.00000 00000 00057 87771 33
8	-0.00000 00000 00006 10226 41	8	-0.00000 00000 00004 70904 90
9	-0.00000 00000 00001 89674 45	9	-0.00000 00000 00000 32532 31
10	0.00000 00000 00000 03278 06	10	0.00000 00000 00000 02342 44
11	0.00000 00000 00000 00564 71	11	0.00000 00000 00000 00095 65
12	-0.00000 00000 00000 00012 91	12	-0.00000 00000 00000 00007 45
13	-0.00000 00000 00000 00000 97	13	-0.00000 00000 00000 00000 13
14	0.00000 00000 00000 00000 03	14	0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 8	r	$E_{r,k}$, k = 9
0	-0.00000 00002 21680 60720 02	0	-0.00000 00000 41980 55790 89
1	-0.00000 00000 46346 89513 21	1	0.00000 00000 89743 74528 40
2	0.00000 00002 23822 07317 47	2	0.00000 00000 45546 98357 54
3	0.00000 00000 07897 35545 06	3	-0.00000 00000 09775 82430 10
4	-0.00000 00000 11219 28958 25	4	-0.00000 00000 02340 53868 72
5	-0.00000 00000 00235 05707 69	5	0.00000 00000 00279 36821 34
6	0.00000 00000 00198 78308 17	6	0.00000 00000 00041 44833 81
7	0.00000 00000 00002 18048 18	7	-0.00000 00000 00003 62585 15
8	-0.00000 00000 00001 71492 48	8	-0.00000 00000 00000 34547 15
9	-0.00000 00000 00000 00308 26	9	0.00000 00000 00000 02647 09
10	0.00000 00000 00000 00035 55	10	0.00000 00000 00000 00153 59
11	-0.00000 00000 00000 00006 31	11	-0.00000 00000 00000 00012 07
12	-0.00000 00000 00000 00002 48	12	-0.00000 00000 00000 00000 37
13	0.00000 00000 00000 00000 04	13	0.00000 00000 00000 00000 04

r	$E_{r,k}$, k = 10	r	$E_{r,k}$, k = 11
0	0.00000 00000 01769 02144 88	0	0.00000 00000 04375 77462 10
1	0.00000 00000 38545 06838 43	1	0.00000 00000 05431 20942 97
2	-0.00000 00000 00653 29497 76	2	-0.00000 00000 04288 74095 01
3	-0.00000 00000 04402 50830 16	3	-0.00000 00000 00649 41793 33
4	0.00000 00000 00025 69456 29	4	0.00000 00000 00219 68452 22
5	0.00000 00000 00126 65203 78	5	0.00000 00000 00018 36896 61
6	-0.00000 00000 00000 92934 20	6	-0.00000 00000 00004 14500 08
7	-0.00000 00000 00001 58474 08	7	-0.00000 00000 00000 21035 45
8	0.00000 00000 00000 01884 50	8	0.00000 00000 00000 03974 60
9	0.00000 00000 00000 01063 97	9	0.00000 00000 00000 00114 58
10	-0.00000 00000 00000 00019 24	10	-0.00000 00000 00000 00022 48
11	-0.00000 00000 00000 00004 25	11	-0.00000 00000 00000 00000 27
12	0.00000 00000 00000 00000 11	12	0.00000 00000 00000 00000 08
13	0.00000 00000 00000 00000 01		

TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu) \ , \ 0 \leq \nu \leq 1 \ .$$

r	$E_{r,k}$, k = 12
0	0.00000 00000 01202 47474 53
1	-0.00000 00000 02020 10084 98
2	-0.00000 00000 01269 13315 16
3	0.00000 00000 00224 65057 78
4	0.00000 00000 00065 53275 44
5	-0.00000 00000 00006 74293 67
6	-0.00000 00000 00001 19434 53
7	0.00000 00000 00000 09413 78
8	0.00000 00000 00000 01052 55
9	-0.00000 00000 00000 00075 22
10	-0.00000 00000 00000 00005 13
11	0.00000 00000 00000 00000 38
12	0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 13
0	-0.00000 00000 00116 71293 14
1	-0.00000 00000 01120 18598 69
2	0.00000 00000 00092 51424 40
3	0.00000 00000 00129 25357 13
4	-0.00000 00000 00004 83236 87
5	-0.00000 00000 00003 78732 17
6	0.00000 00000 00000 10936 36
7	0.00000 00000 00000 04867 92
8	-0.00000 00000 00000 00139 86
9	-0.00000 00000 00000 00033 85
10	0.00000 00000 00000 00001 08
11	0.00000 00000 00000 00000 14
12	-0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 14
0	-0.00000 00000 00160 76526 03
1	-0.00000 00000 00052 16605 52
2	0.00000 00000 00161 99991 75
3	0.00000 00000 00006 69416 97
4	-0.00000 00000 00008 43079 67
5	-0.00000 00000 00000 17039 12
6	0.00000 00000 00000 16088 04
7	0.00000 00000 00000 00123 39
8	-0.00000 00000 00000 00155 23
9	0.00000 00000 00000 00000 36
10	0.00000 00000 00000 00000 88
11	-0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 15
0	-0.00000 00000 00021 02319 23
1	0.00000 00000 00118 79215 58
2	0.00000 00000 00023 75611 36
3	-0.00000 00000 00013 62902 86
4	-0.00000 00000 00001 22668 07
5	0.00000 00000 00000 40915 14
6	0.00000 00000 00000 02134 75
7	-0.00000 00000 00000 00555 34
8	-0.00000 00000 00000 00016 64
9	0.00000 00000 00000 00004 22
10	0.00000 00000 00000 00000 06
11	-0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 16
0	0.00000 00000 00015 71000 80
1	0.00000 00000 00028 26021 77
2	-0.00000 00000 00015 42055 63
3	-0.00000 00000 00003 32009 33
4	0.00000 00000 00000 81110 74
5	0.00000 00000 00000 09615 41
6	-0.00000 00000 00000 01606 73
7	-0.00000 00000 00000 00118 09
8	0.00000 00000 00000 00016 53
9	0.00000 00000 00000 00000 74
10	-0.00000 00000 00000 00000 10

r	$E_{r,k}$, k = 17
0	0.00000 00000 00005 16156 99
1	-0.00000 00000 00010 75033 89
2	-0.00000 00000 00005 44377 21
3	0.00000 00000 00001 22978 90
4	0.00000 00000 00000 28402 63
5	-0.00000 00000 00000 03786 52
6	-0.00000 00000 00000 00528 39
7	0.00000 00000 00000 00054 09
8	0.00000 00000 00000 00004 81
9	-0.00000 00000 00000 00000 44
10	-0.00000 00000 00000 00000 02

TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu) \ , \ 0 \leq \nu \leq 1 \ .$$

r	$E_{r,k}$, k = 18
0	-0.00000 00000 00001 41637 33
1	-0.00000 00000 00005 16350 90
2	0.00000 00000 00001 35183 84
3	0.00000 00000 00000 60287 46
4	-0.00000 00000 00000 07203 72
5	-0.00000 00000 00000 01784 52
6	0.00000 00000 00000 00148 78
7	0.00000 00000 00000 00023 13
8	-0.00000 00000 00000 00001 63
9	-0.00000 00000 00000 00000 16
10	0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 19
0	-0.00000 00000 00000 86469 63
1	0.00000 00000 00000 94870 21
2	0.00000 00000 00000 89618 69
3	-0.00000 00000 00000 10814 39
4	-0.00000 00000 00000 04707 93
5	0.00000 00000 00000 00343 60
6	0.00000 00000 00000 00089 79
7	-0.00000 00000 00000 00005 21
8	-0.00000 00000 00000 00000 86
9	0.00000 00000 00000 00000 05

r	$E_{r,k}$, k = 20
0	0.00000 00000 00000 13296 15
1	0.00000 00000 00000 81839 70
2	-0.00000 00000 00000 12253 81
3	-0.00000 00000 00000 09552 00
4	0.00000 00000 00000 00663 96
5	0.00000 00000 00000 00286 14
6	-0.00000 00000 00000 00014 42
7	-0.00000 00000 00000 00003 80
8	0.00000 00000 00000 00000 17
9	0.00000 00000 00000 00000 03

r	$E_{r,k}$, k = 21
0	0.00000 00000 00000 13621 70
1	-0.00000 00000 00000 09393 28
2	-0.00000 00000 00000 14033 00
3	0.00000 00000 00000 01067 32
4	0.00000 00000 00000 00740 94
5	-0.00000 00000 00000 00035 14
6	-0.00000 00000 00000 00014 33
7	0.00000 00000 00000 00000 57
8	0.00000 00000 00000 00000 14
9	-0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 22
0	-0.00000 00000 00000 01546 44
1	-0.00000 00000 00000 12886 32
2	0.00000 00000 00000 01389 77
3	0.00000 00000 00000 01506 04
4	-0.00000 00000 00000 00076 51
5	-0.00000 00000 00000 00045 44
6	0.00000 00000 00000 00001 73
7	0.00000 00000 00000 00000 61
8	-0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 23
0	-0.00000 00000 00000 02174 39
1	0.00000 00000 00000 01302 27
2	0.00000 00000 00000 02238 56
3	-0.00000 00000 00000 00148 48
4	-0.00000 00000 00000 00118 62
5	0.00000 00000 00000 00004 97
6	0.00000 00000 00000 00002 31
7	-0.00000 00000 00000 00000 08
8	-0.00000 00000 00000 00000 02

TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(z) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(z) \ , \ 0 \leq \nu \leq 1$$

r	$E_{r,k}$, k = 24	r	$E_{r,k}$, k = 25
0	0.00000 00000 00000 00263 37	0	0.00000 00000 00000 00355 89
1	0.00000 00000 00000 02085 63	1	-0.00000 00000 00000 00271 33
2	-0.00000 00000 00000 00240 75	2	-0.00000 00000 00000 00367 41
3	-0.00000 00000 00000 00244 23	3	0.00000 00000 00000 00031 26
4	0.00000 00000 00000 00013 28	4	0.00000 00000 00000 00019 51
5	0.00000 00000 00000 00007 40	5	-0.00000 00000 00000 00001 03
6	-0.00000 00000 00000 00000 30	6	-0.00000 00000 00000 00000 38
7	-0.00000 00000 00000 00000 10	7	0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 26	r	$E_{r,k}$, k = 27
0	-0.00000 00000 00000 00060 77	0	-0.00000 00000 00000 00058 49
1	-0.00000 00000 00000 00344 51	1	0.00000 00000 00000 00069 24
2	0.00000 00000 00000 00057 78	2	0.00000 00000 00000 00060 72
3	0.00000 00000 00000 00040 43	3	-0.00000 00000 00000 00008 05
4	-0.00000 00000 00000 00003 16	4	-0.00000 00000 00000 00003 23
5	-0.00000 00000 00000 00001 23	5	0.00000 00000 00000 00000 26
6	0.00000 00000 00000 00000 07	6	0.00000 00000 00000 00000 06
7	0.00000 00000 00000 00000 02		

r	$E_{r,k}$, k = 28	r	$E_{r,k}$, k = 29
0	0.00000 00000 00000 00015 56	0	0.00000 00000 00000 00009 18
1	0.00000 00000 00000 00056 10	1	-0.00000 00000 00000 00018 15
2	-0.00000 00000 00000 00015 27	2	-0.00000 00000 00000 00009 62
3	-0.00000 00000 00000 00006 60	3	0.00000 00000 00000 00002 12
4	0.00000 00000 00000 00000 83	4	0.00000 00000 00000 00000 51
5	0.00000 00000 00000 00000 20	5	-0.00000 00000 00000 00000 07
6	-0.00000 00000 00000 00000 02	6	-0.00000 00000 00000 00000 01

TABLE 3 (Concluded)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z) \ , \ z \geq 8 \ ,$$

$$M_k(z) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(z) \ , \ 0 \leq \nu \leq \Gamma$$

r	$E_{r,k}$, k = 30
0	-0.00000 00000 00000 00003 95
1	-0.00000 00000 00000 00008 40
2	0.00000 00000 00000 00003 95
3	0.00000 00000 00000 00000 99
4	-0.00000 00000 00000 00000 21
5	-0.00000 00000 00000 00000 03

r	$E_{r,k}$, k = 31
0	-0.00000 00000 00000 00001 24
1	0.00000 00000 00000 00004 55
2	0.00000 00000 00000 00001 32
3	-0.00000 00000 00000 00000 53
4	-0.00000 00000 00000 00000 07
5	0.00000 00000 00000 00000 02

r	$E_{r,k}$, k = 32
0	0.00000 00000 00000 00000 94
1	0.00000 00000 00000 00000 97
2	-0.00000 00000 00000 00000 96
3	-0.00000 00000 00000 00000 11
4	0.00000 00000 00000 00000 05

r	$E_{r,k}$, k = 33
0	0.00000 00000 00000 00000 09
1	-0.00000 00000 00000 00001 05
2	-0.00000 00000 00000 00000 11
3	0.00000 00000 00000 00000 12
4	0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 34
0	-0.00000 00000 00000 00000 21
1	-0.00000 00000 00000 00000 01
2	0.00000 00000 00000 00000 21
3	0.00000 00000 00000 00000 00
4	-0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 35
0	0.00000 00000 00000 00000 02
1	0.00000 00000 00000 00000 22
2	-0.00000 00000 00000 00000 02
3	-0.00000 00000 00000 00000 03

r	$E_{r,k}$, k = 36
0	0.00000 00000 00000 00000 04
1	-0.00000 00000 00000 00000 04
2	-0.00000 00000 00000 00000 04

r	$E_{r,k}$, k = 37
0	-0.00000 00000 00000 00000 01
1	-0.00000 00000 00000 00000 04
2	0.00000 00000 00000 00000 01

r	$E_{r,k}$, k = 38
0	-0.00000 00000 00000 00000 01
1	0.00000 00000 00000 00000 02
2	0.00000 00000 00000 00000 01

$E_{r,k}$, k = 39

The coefficients for k = 39 are all zero to 22 decimals.

r	$E_{r,k}$, k = 40
0	0.00000 00000 00000 00000 00
1	-0.00000 00000 00000 00000 01

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

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